Tarea 5

Estabilidad

Entregar el Lunes 10 de Noviembre

Problema 1. *Hirsh Smale Página 215 problema 1*

1. **Find the differential equations for the network in Fig. D, where the resistor is voltage controlled, that is, the resistor characteristic is the graph of a C^1 function $g: \mathbf{R} \to \mathbf{R}$, $g(v_R) = i_R$.**

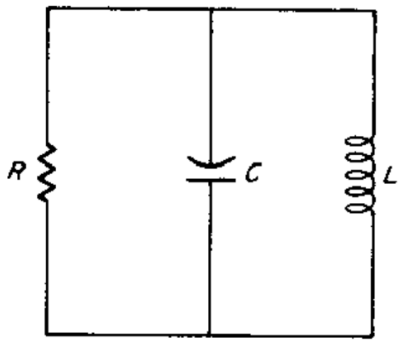

FIG. D

Problema 2. *Hirsh Smale Página 215 problema 2*

2. **Show that the LC circuit consisting of one inductor and one capacitor wired in a closed loop oscillates.**

Problema 3. *Hirsh Smale Página 226 problema 3*

3. (Hartman [9, Chapter 7, Theorem 10.2]) Find the phase portrait of the following differential equation and in particular show there is a unique nontrivial periodic solution:

$$x' = y - f(x),$$
$$y' = -g(x),$$

where all of the following are assumed:
(i) f, g are C^1;
(ii) $g(-x) = -g(x)$ and $xg(x) > 0$ for all $x \neq 0$;
(iii) $f(-x) = -f(x)$ and $f(x) < 0$ for $0 < x < a$;
(iv) for $x > a$, $f(x)$ is positive and increasing;
(v) $f(x) \to \infty$ as $x \to \infty$.
(*Hint*: Imitate the proof of the theorem in Section 3.)

Problema 4. *Hirsh Smale Página 226 problema 5*

5. Consider the equation

$$x' = \mu\big(y - (x^3 - x)\big), \qquad \mu > 0,$$
$$y' = -x.$$

It has a unique nontrivial periodic solution γ_μ by Problem 3. Show that as $\mu \to \infty$, γ_μ tends to the closed curve consisting of two horizontal line segments and two arcs on $y = x^3 - x$ as in Fig. I.