Optimal Motion Strategies Based on Critical Events to Maintain Visibility of a Moving Target

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***Abstract*— In this paper, we consider the surveillance problem of maintaining visibility at a fixed distance of a mobile evader using a mobile robot equipped with sensors.**

Optimal motion for the target to escape is found. Symmetrically, an optimal motion strategy for the observer to always maintain visibility of the evader is determined.

The optimal motion strategies proposed in this paper are based on critical events. The critical events are defined with respect to the obstacles in the environment.

I. Introduction

In this paper, we consider the surveillance problem of maintaining visibility at a fixed distance of a mobile evader (the target) using a mobile robot equipped with sensors (the observer).

We address the problem of maintaining visibility of the target in the presence of obstacles. We assume that obstacles produce both motion and visibility constraints. We consider that both the observer and the target have bounded velocity. We assume that the pursuer can react instantaneously to evader motion.

This problem has two important aspects. The first one is to find an optimal motion for the target to escape and symmetrically to determine the optimal strategy for the observer to always maintain visibility of the evader. The second aspect is to determine the necessary and sufficient conditions for the existence of a solution.

In this paper we address the first aspect of the problem. That is, determine the optimal motion strategy, which corresponds to define how the evader and pursuer should move. We have numerically found which are the optimal controls (velocity vectors) that the target has to apply to escape observer surveillance. We have also found which are the optimal controls that the observer must applied to prevent the escape of the target.

In our previous research, we have considered variations of the problem of maintaining visibility of a moving evader with a mobile robot. In [13] we considered the case where there is a delay but no velocity bounds for the observer. In [14] we addressed the case in which there is no delay, but the observer's velocity is bounded. We define necessary conditions for the existence of a surveillance strategy and give an algorithm that generates surveillance strategies. Additionally, we provide the observer control to prevent the target escape for the case of a straight line target trajectory.

As in [14], here we consider the case of no delay and both observer and target bounded speed, but *in this paper, we provide optimal controls for the target to escape and we propose an observer motion strategy to prevent target escaping*.

Geometric reasoning and optimal control techniques are the tools to model the problem and find appropriate motion strategies.

A. Previous Work

Our problem is related to pursuit-evasion games. A great deal of previous research exists in the area of pursuit and evasion, particularly in the area of dynamics and control in the free space [5], [10], [1]. These works typically do not take into account constraints imposed on the observer motion due to the existence of obstacles in the workspace, nor visibility constraints that arise due to occlusion.

The pursuit-evasion problem is often framed as a problem in non cooperative dynamic game theory [1]. A pursuit-evasion game can be defined in several manners.

One of them consists in *finding* an evasive target with one or more mobile pursuers that sweep the environment so that the target does not eventually sneak into an area that has already been explored. Deterministic [17], [21], [4], [20] and probabilistic algorithms [22], [6] have been proposed to solve this problem. The pursuers could also be interested to actually "catch" the evaders, that is, move to a contact configuration or closer than a given distance [10].

As mentioned above, our problem is related to the problems of pursuit-evasion. However, the previous problems are not the same as ours. In this paper, we assume that initially the pursuer can establish visibility with the evader. The problem consists in determining a motion pursuer strategy to always maintain the visibility between the evader and the pursuer. We call such a task target tracking.

The target tracking problem has often been attacked with a combination of vision and control techniques (see, e.g., [15], [3], [8]). Purely control approaches, however, do not take into account the existence of obstacles in the the environment. The basic question that must be answered is *where should the robot observer move in order to maintain visibility of a target moving in a cluttered workspace?* Both visibility and motion obstructions must be taken into account, and thus, a pure visual servoing technique can fail because it ignores the global geometry of the workspace.

Previous works have also studied the motion planning problem for maintaining visibility of a moving evader (target tracking) in the presence of obstacles. Game theory is proposed in [11] as a framework to formulate the tracking problem and an online algorithm is presented.

In [2], an algorithm is presented which operates by maximizing the probability of future visibility of the target. This algorithm is also studied with more formalism in [11]. This technique was tested in a Nomad 200 mobile robot with relatively good results. However, the probabilistic model assumed by the planner was often too simplistic, and accurate models are difficult to obtain in practice.

The approach presented in [12] computes a motion strategy by maximizing the *shortest distance to escape* —the shortest distance the target needs to move in order to escape the observer's visibility region. In this work the targets are assumed to move unpredictable, and the distribution of obstacles in the workspace is assumed to be known in advance. This planner has been integrated and tested in a robot system which includes perceptual and control capabilities. The approach has also been extended to maintain visibility of two targets using two mobile observers.

In [7], a technique is proposed to track a target without the need of a global map. Instead, a range sensor is used to construct a local map of the environment, and a combinatoric algorithm is then used to compute a differential motion for the observer at each iteration.

More recently, some works have considered the problem of maintaining visibility of several targets with multiple robots. In [12] an algorithm is proposed to maintain visibility of two evaders with two pursuers. In this approach, there is no predetermined assignment of a given target to a given observer. At any instant in time, the two observers locate themselves so as to maximize the distance to escape required by either of the targets.

In [16] a method is proposed to accomplish this task in uncluttered environments. The objective is to minimize the total time in which targets escape observation by some robot team member. In [9] an approach is proposed to maintain visibility of several targets using mobile and static sensors. A metric for measuring the degree of occlusion, based on the average mean free path of a random line segment is used.

The problem of planning observer's motions to maintain visibility of a moving target has received a good deal of attention in the motion planning community over the last years. Several techniques have been reported in the literature, and a variety of strategies have been proposed to perform the tracking. However, the optimal motion strategy for the target to escape in the presence of obstacles and, the optimal observer motion response (for any target trajectory) has never been found before. To give these optimal motion polices is the goal of this paper.

II. Problem definition

The target and the observer are represented as points. The visibility between the target and the observer is represented as a line segment and it is called the rod (or bar). This rod is emulating the visual sensor capabilities of the observer. The constant rod length is modeling a fixed sensor range.

We address the problem of maintaining visibility of the target in the presence of obstacles. The obstacles are modeled with polygonal barriers. We assume that the observer is provided with a map of the environment.

Violation of the visibility constraint corresponds to collision of the rod with an obstacle in the environment. The target controls the rod origin (x, y) and the observer controls the rod's orientation θ and must compensate to maintain a fixed rod length L.

We are assuming that the evader is antagonist, hence, it will not cooperate with the target either helping it to maintain visibility or by inaction. If the target has the opportunity to escape, then it will take the required action to do it. The target can defeat the observer by hiding behind an obstacle (breaking the rod with a vertex), by making the observer collide with and obstacle (a segment or a vertex) or by preventing the observer from being at the required fixed distance.

The target moves continuously, its global trajectory is unknown but its maximal speed is known. We are assuming a feedback control scheme where the target velocity is measured (or reported) without delay. Symmetrically, we assume that the target knows the observer velocity vector as soon as the observer moves (without delay). Both observer and target are limited to move with bounded speed. Both observer and target are holonomic robots.

The optimal target and observer motion strategies are defined as the ones that give the quantitative conditions to prevent the target from escaping. This requires to determine the last moment (critical event) -with respect to the obstacles- when the observer must start changing the rod configuration before it is too late.

III. Problem Modeling

We work at the frontiers of computational geometry algorithms and control algorithms. The originality and the strength of the work is to bring together both aspects.

A. Dealing with obstacles

We are able to express the constraints on the observer dynamics (velocity bounds and kinematics constraints) geometrically, as a function of the geometry of the workspace and the surveillance distance.

In order to maintain surveillance, it is necessary that the line segment connecting the pursuer and evader not intersect any obstacle in the environment (this would result in occlusion of the evader).

Our approach consists in partitioning the configuration space and the workspace in non-critical regions separated by critical curves [19], [13], [14]. These curves bound forbidden rod configurations [13]. These rod configurations are forbidden either because they generate a violation of the visibility constraint (corresponding to a collision of the rod with an obstacle in the environment [13]) or because they require the observer to move with speed greater than its maximum [14].

In order to avoid a forbidden rod configuration, the pursuer must change the rod configuration to prevent the target to escape. We call this pursuer motion the rotational motion [14].

This type of motion will be finished either when the observer brings the rod to a configuration that avoids an escapable cell [13], when the observer reaches and aspect graph line [18] associated to a reflex vertex or, when the observer is able to move the rod in contact with an obstacle [14].

If the observer has bounded speed then the rotational motion has to be started far enough for any forbidden rod configuration. The pursuer must have enough time to change the rod configuration before the evader brings the rod to a forbidden one. There are critical events that tell the pursuer to start changing the rod configuration before it is too late. These critical events depend on the geometry of the environment, the initial location of the evader x, y, the relative configurations of the pursuer and evader θ, the final rod configuration that prevents the evader to escape and the maximal observer and evader speeds. The critical events signal the observer to start the rotational motion with enough time for preventing that the target reaches an escape point.

In [14] we define an escape point as a point on a critical curve associated to an escapable cell [13], or a point in a region bounding an obstacle. This region is bounding either a reflex vertex (those with interior angle larger than π) or segment of the polygonal workspace.

Merely reaching an escape point does not guarantee that the evader can escape the surveillance. An escape point is a point from which the evader may escape for *some set* of observer positions (i.e., for some set of configurations, (x, y, θ) of the rod). Thus, when the evader nears an escape point, the observer must take action to ensure future visibility of the evader. Since the observer has bounded velocity, it must react before the escape point is reached by the evader. For more details see [14].

Similarly, we denote by D the minimal distance from an escape point such that, if the evader is further than D from the escape point, the observer will have sufficient time to react and prevent escape. Thus, it is only when the evader is nearer than D to an escape point that the observer must take special care. Thus, the critical events are to D distance from the escape points.

In order to better clarify our description, we present one simple example. This example shows a convex corner (see figure 1). Solid lines indicate the critical curves at l distance from the obstacles and dashed lines indicate the critical events as a function of the distance from the first set of critical curves. The dot labeled (T) indicates the target and the dot labeled (O) the observer. A rod of length l is indicated with a segment finished with T and O labels. The graph in the figure indicates cell adjacency in the configuration space (see [13]).

When the evader is approaching the corner, the observer must rotate around the evader to change the rod configuration, otherwise the evader can violate the visibility constrain. This can be by making the rod collide with a obstacle or by forcing the pursuer to move with speed greater than its maximum (see [14]).

The observer can choose to go to anywhere in region $R3$. The shorter rotation in this case is moving just to the border of $R3$.

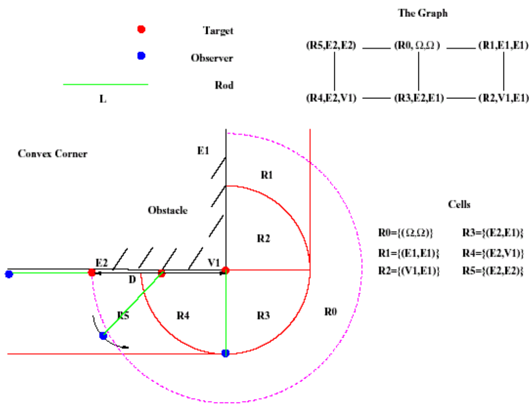

Fig. 1. Convex Corner

Therefore, if the rod is in a non-admissible configuration then the target can get further from the observer than the fixed surveillance distance.

IV. Optimal Target and Observer Motions

Take the global Cartesian axis to be defined such that the origin is the target's initial position, and the x-axis is the line connecting the target's initial position and the escape point. The target and observer velocities are saturated at V_t and V_o respectively, and because the rod length must be fixed at all times, the relative velocity V_{ot} must be perpendicular to the rod. This information yields the following velocity vector diagram (see figure 2).

The law of cosines can be used to determine $\|Vot\|$.

$$V_o{}^2 = V_t{}^2 + V_{ot}{}^2 - 2V_tV_{ot}\cos(\alpha + \theta + \frac{\pi}{2}) \quad (1)$$

After solving the equation and some simplification, the final result is:

$$\|Vot\| = -V_t\sin(\alpha + \theta) \pm \sqrt{V_o{}^2 - V_t{}^2\cos^2(\alpha + \theta)} \quad (2)$$

The rate of change of theta can be found easily as:

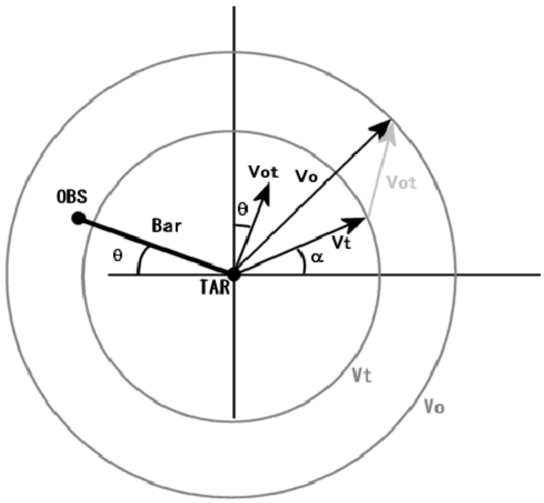

Fig. 2. velocity vector diagram

$$\frac{d\theta}{dt} = \frac{Vot}{L} = \frac{-V_t \sin(\alpha+\theta) \pm \sqrt{{V_o}^2 - {V_t}^2 \cos^2(\alpha+\theta)}}{L} \tag{3}$$

Because the boundary conditions of the geometry are defined in terms of x, a more useful derivative would be:

$$\frac{d\theta}{dx} = \frac{d\theta}{dt}(\frac{dx}{dt})^{-1} = \frac{-R\sin(\alpha+\theta) \pm \sqrt{1 - R^2\cos^2(\alpha+\theta)}}{L\cos(\alpha)} \tag{4}$$

Where $R = \frac{V_t}{V_o} < 1$

The optimal path for the target can be defined in two equivalent ways. One formulation, given a starting point at the origin, an escape point at (x_1, y_1), and an initial angle θ_1, the optimal path $\alpha(x), x \in [0, x_1]$ should minimize the amount of angle that the observer can make up in its rotation up until the target reaches the escape point. The alternate formulation would be to find for a given initial rod angle θ_0, the maximum distance to an escape point x_1 such that the final rod configuration is at a specified final angle θ_1 and the corresponding target motion $\alpha(x)x \in [0, x_1]$. The proposed solution method gives the solution to both formulations. First, define a state space model with boundary conditions.

The natural representation would be:

$$\frac{d\theta}{dx} = \frac{-R\sin(\alpha+\theta) \pm \sqrt{1 - R^2\cos^2(\alpha+\theta)}}{L\cos(\alpha)} = f_1 \tag{5}$$

$$\frac{dy}{dx} = V_t \sin\alpha = f_2 \tag{6}$$

$$\theta(0) = \theta_0, \theta(x_1) = \theta_1, y(0) = y(1) = 0 \tag{7}$$

To maximize x_1, the appropriate cost function is:

$$V = -\int_0^{x_1} dx \tag{8}$$

The optimal control problem can be stated using 4 conditions:

1) There exists two functions of x, p_1 and p_2 such that $\alpha^* = argmin[p_1 f_1 + p_2 f_2 - 1]$ is satisfied pointwise for all x.
2) The state vector satisfies the state equations and 4 boundary conditions above
3) The final value x_1 satisfies $p_1(x_1)f_1(x_1) + p_2(x_1)f_2(x_1) - 1 = 0$
4) The state equations for are given by:

$$\frac{\partial p_1}{\partial x} = -\frac{\partial f_1}{\partial \theta} p_1 = \frac{\cos(\alpha-\theta)\frac{R^2\cos(\alpha+\theta)\sin(\alpha+\theta)}{\sqrt{1-R^2\cos^2(\alpha+\theta)}}}{L\cos\alpha} p_1 \tag{9}$$

$$\frac{\partial p_2}{\partial x} = -\frac{\partial f_2}{\partial y} p_2 \equiv 0 \rightarrow p_2(x) = P_2 \quad (constant) \tag{10}$$

If we choose p_2 as zero, the minimization condition 1 simplifies to

$$\alpha^*(x) = argmin[p_1 f_1 + p_2 f_2 - 1] \rightarrow \frac{\partial}{\partial \alpha}[p_1 f_1 + p_2 F_2 - 1] = 0$$

$$\rightarrow \frac{\partial f_1}{\partial \alpha} = 0 \tag{11}$$

The last step can be inferred because for nonzero initial p_1, p_1 will not reach equilibrium at zero.

A strategy to generate the solution to the boundary value problem is as follows:

Generate the θ-Minimizing Curve by integrating the observer and target positions forward in x, choosing the minimizer α^* at every step Δx.

Select two points on the curve and let the line through them represent the new x-axis. The two points can be chosen so that the initial and final angle conditions are satisfied (as measured with respect to the new x-axis), and the optimal path is then the section of the θ-Minimizing Curve connecting the two points. The distance between the two points is x_1, the critical distance D to the escape point for the given initial angle and critical escape angle.

The alternate problem, finding the minimum angle turned given an initial angle and distance D to an escape point, can be solved similarly. Select two points on the curve such that the distance between the two is D, and define the new x-axis as the line between them. If the initial angle condition is satisfied with respect to the new axis, then the minimum final angle is given by the angle at the second endpoint. The optimal path is again the section of the curve between the two points.

Since we minimized over all α in taking a step dx, minimization condition 1 is satisfied . In the new rotated x-axis, condition 2 (boundary conditions) are satisfied. Conditions 3 and 4 can be satisfied since we theoretically can always find some value of $p_1(x_0)$ that will satisfy this condition just by integrating the equation of condition 4 backwards in time. Therefore we are certain to have a solution to the optimal control problem. Moreover, since $f_1(\alpha)$ has a unique minimizer, this is the unique solution. It is graphically illustrated below (see figure 3).

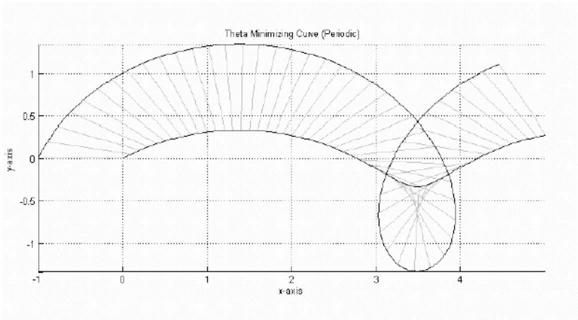

Fig. 3. Θ Minimizing Curve

Figure 4 shows the general case when the initial distance to the escape point is fixed. We wish to find the optimal path that the target should take to the escape point, and what the critical escape angle is $L = D = 1, \theta_0 = 15\,\text{deg}, V_o = 2, V_t = 1$

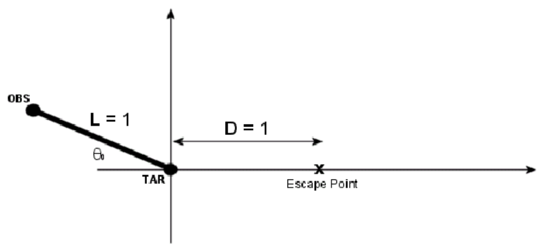

Fig. 4. Fixed distance to the escape point

Searching along the θ minimizing curve, we can find two points which satisfy the initial condition and distance to escape criteria.

The final angle is found to be 81.1 degrees. For corners whose sharpness exceeds this angle, the target will be able to escape. Otherwise, the observer will be able to track the target (see figures 5 and 6).

In some cases, instead of a single escape point, there is an escape line to reach. For example, in the situation below (see figure 7), if the target can reach cell II (escapable cell, see [13]) before the observer can rotate θ to 180 $\deg$, the target escapes. Because there is no constraint in the y-direction now, the solution is much simpler. In this case, it is simply the θ minimizing curve generated with the origin at the target initial position and x-axis perpendicular to the escape line. The initial condition is the observer's initial angle. The point where the trajectory hits the escape line is the potential escape point, and we can also find the final angle, determining success or failure of escape.

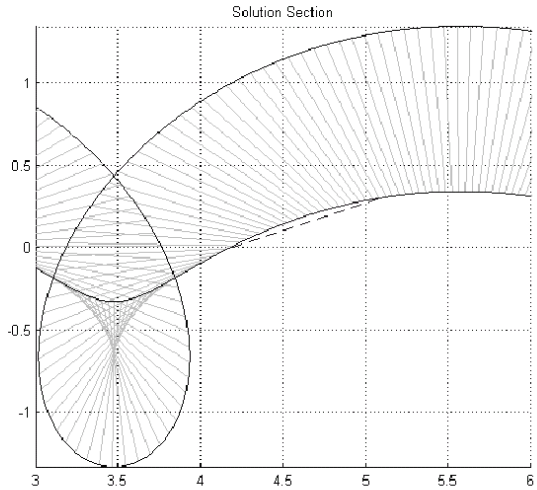

Fig. 5. Distance to escape point

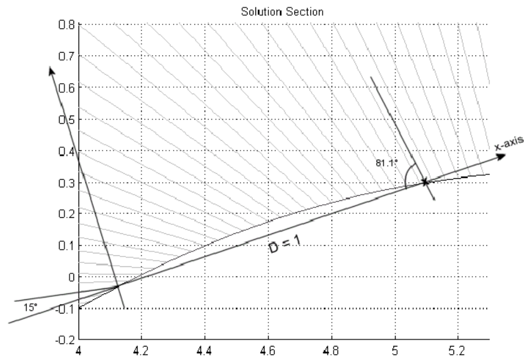

Fig. 6. Zoom: Distance to escape point

Note that a straight line path minimizes the time to reach an escape point, but the optimal target path to escape is a different curve (see figure 3). This happens because there are two different times that must be considered. The time taken for the target to reach the escape point and the time taken for the observer to change the rod configuration. Because of the kinematic constraints (bounded speeds and fixed surveillance distance), there is a trade-off between minimizing the time taken for the target to reach the escape point and maximizing the time taken for the observer to change the rod configuration. Therefore, the optimal target path is the one that minimizes the amount of angle that the observer can make in its rotation up until the target reaches the escape point.

The optimal observer motion strategy consists in once a critical event is detected then the observer must saturate its speeding and, start its rotation around the target.

In fact, there are several motions that will keep the observer at a constant distance from the target. The simplest is to apply the same motion vector as the target. Another option is to always move in the direction of the target. However, the only

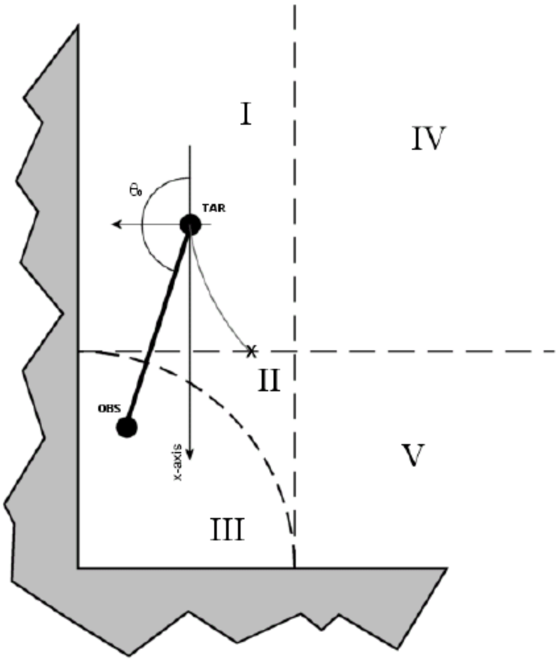

Fig. 7. Escapable cell case

motion in which the observer can change the bar to a particular final orientation independently of the target trajectory consists in applying the same velocity vector to the observer as the one that the target applies and an additional vector to get an observer rotation around the target.

Therefore, this is the only strategy that does not require target cooperation (antagonistic target). Note that the target trajectory can influence the rate of change, but not the initial and final orientations.

For the above reason, the movement policy for the observer is the same regardless of the actual target trajectory. Because of the constant distance constraint, the observer can only move perpendicularly to the rod (rotate around the target).

V. Conclusions and future work

This paper solved the game of degree [10], [1] of maintaining visibility at a fixed distance of a moving target with a mobile robot in the presence of obstacles.

Solving this game corresponds to find the quantitative conditions to prevent the target from escaping. This problem is solved by determining the last moment (critical event) -with respect to the obstacles- when the observer must start changing the rod configuration before is too late.

The critical events are defined according to the optimal (maximal and minimal) target and observer control polices (which will correspond to target and observer optimal trajectories).

Maintaining visibility at a *constant distance* of a evader with a mobile robot is one (amount others) of the possible problem formulations. This formulation allows us to get a well defined problem. However its main drawback is that this task requires very accurate control over the observer.

Another problem formulation is to relax the constraint and maintain visibility at a variable distance. As future work, we want to investigate that problem. We also want to investigate the case of a non-holonomic pursuer and a holonomic evader.

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