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On the Value of Information in a Differential Pursuit-Evasion Game

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Abstract— **In this paper, we address the pursuit/evasion problem of capturing an omnidirectional evader using a Differential Drive Robot (DDR) in an obstacle-free environment. The goal of the evader is to keep the pursuer farther than the capture distance for as long as possible and for the pursuer the goal is to capture the evader as soon as possible. In [1] an open-loop time-optimal strategy is proposed for this pursuit/evasion problem. In [2] a state feedback-based time-optimal motion policy for the DDR is provided. The time-optimal strategies obtained in [1] are in Nash equilibrium, meaning that any unilateral deviation of a player from the optimal strategies does not provide it a benefit in its payoff. However, Nash equilibrium does not tell if one player deviates from its optimal policy then, does there exist a new strategy for the other player that can take advantage of such deviation? If so, which is the required information to improve the payoff compared with the worst case scenario? In this paper we address those questions, analysing the scenario in which the players deviate from their optimal controls. We show that when the evader deviates from its optimal speed there are cases where there exists a new pursuer motion strategy that reduces the time to capture the evader. The shown cases where the time to capture the evader is reduced require more information about the evader's state. Nevertheless, there are also cases in which despite the availability of new information, the pursuer must stick to the worst case strategy, otherwise it cannot capture the evader.**

I. Introduction

In this paper, we address the pursuit-evasion problem of capturing an omnidirectional evader using a Differential Drive Robot (DDR) in an obstacle-free environment. At the beginning of this game, the evader is at a distance $L > l$ (the capture distance) from the pursuer. The goal of the evader is to keep the pursuer farther than this capture distance for as long as possible. The goal of the pursuer is to capture the evader as soon as possible.

In previous work, in [1] the authors have proposed a partition of the playing space into mutually disjoint regions where time optimal strategies of the players are well established. The time-optimal strategies obtained in [1] are in Nash equilibrium. In [1], the proposed strategies are in open-loop. Later, in [2], the authors provided a state feedback-based time-optimal motion policy for the DDR. This later result was achieved by estimating the state of the evader based on images using the 1D trifocal tensor.

In [2], the planning stage makes use of the partition presented in [1]. But the authors analyse the situation, in which in the execution stage, the pursuer applies its optimal policy in open loop (without evader state feedback), and the evader follows a suboptimal policy. In particular, in [2], it has been shown that if the pursuer executes its optimal policy in open loop then the pursuer might not be able to capture the evader. Hence, in the case of an unpredictable evader, it is crucial for the pursuer to execute a state feedback-based motion policy based on the evader state. Nevertheless, in [2], the authors do not analyse whether or not there exists a pursuer strategy that reduces the time of capture when the evader deviates from its optimal motion policy.

In this paper, we analyse this last scenario, in which the players deviate from their optimal controls. In Nash equilibrium, any unilateral deviation of a player from the optimal strategies does not provide it a benefit in its payoff. However, Nash equilibrium does not tell if one player deviates from its optimal policy then, does there exist a new strategy for the other player that can take advantage of such deviation? If so, which is the required information to improve the payoff compared with the worst case scenario? In this paper, we address those questions.

Our system model considers that the controls for the DDR pursuer are the wheels angular velocities, and for the omnidirectional evader the speed and the motion direction. In particular, in this work, we analyse the case where the evader deviates from its optimal speed and the effects of this deviation over the pursuer strategy. We show that when the evader deviates from its optimal speed there are cases where there exists a new pursuer motion strategy that reduces the time to capture the evader compared with the worst case analysis. The shown cases where the pursuer payoff is improved require more information about the state of the evader, including now the instantaneous evader speed. Nevertheless, there are also cases in which despite the availability of new information, namely the instantaneous evader speed, the pursuer cannot improve its payoff, as it must apply the worst case strategy, otherwise the capture of the evader is not longer guaranteed.

II. Related Work

The problem addressed in this paper is related to pursuit-evasion games. There has been a considerable amount of research in the area of pursuit and evasion, particularly in the area of control [3], [4], [6]. The pursuit-evasion problem can be framed as a problem in noncooperative dynamic game theory [6].

A pursuit-evasion game can be defined in several ways. One variant considers one or more pursuers, which are given the task of *finding* an evader in an environment with obstacles

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[7], [8], [10], [11]. A recent survey of this kind of problem is presented in [12].

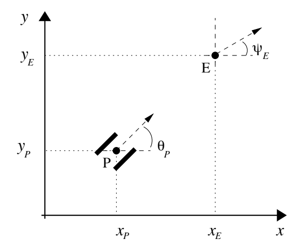
Other variant consists of *maintaining visibility of a moving evader* also in an environment with obstacles [13], [14], [15], [16], [17], [20]. Game theory is proposed in [13] as a framework to formulate the tracking problem, and an online algorithm is presented. In [20], the authors address the problem of maintaining visibility of the evader as a *game of degree* (i.e. the emphasis is over the optimization of a given criterion and not over the problem of deciding who is the winner). The pursuer and the evader are omnidirectional (holonomic) systems. In [21], the problem of maintaining visibility of a moving evader is addressed as a *game of kind* (deciding which player wins). Again, both the pursuer and the evader are omnidirectional systems.

Similar to this work, in [17] the author deals with the information required to achieve the task. In that work, a robot has to track an unpredictable target. The robot's sensors obtain general information about the target's movements, but avoiding that detailed information about the target's position is accessible to an agent that can damage the target using it, preserving in that way the target's privacy. As in the presented work, in [17] the author is also interested in the value of information, nevertheless, in [17] the information is addressed to preserve privacy, while in our work our interest in information is focused on obtaining optimality in the task given to the robot.

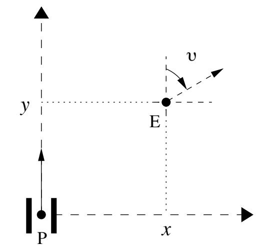
A third variant of pursuit-evasion problem consists in giving to the pursuer the goal *to capture the evader* [3], [4], that is, move to a contact configuration, or closer than a given distance. The work presented in this paper corresponds to this third variant. Other related problems are the lady in the lake [6] and the lion and the man [18], [19]. In the lady in the lake problem, there is a circular lake where a lady is swimming with a maximum speed v_l, and there is a man that is in the side of the lake and runs along the shore with a maximum speed v_m; the man cannot enter the lake and the lady wants to leave the lake. The man runs with a larger speed than the one of the lady in the lake ($v_l < v_m$). The man needs to capture the lady as soon as she reaches the shore, since on land she runs faster than him. In the lion and the man problem, the players move in a circular arena, both players have the same motion capabilities, the lion wants to capture the man and the man wants to avoid the capture.

In the same vein, in [9] the authors address a pursuit-evasion game in a graph called the cops and robbers game. The cops win the game if they can move to the robber's vertex. Alike the presented work, the authors investigate the role of the available information, however, in [9] the authors start from a base case where the players "see" each other at all times and then the authors reduce the visibility range of the players, while in the presented work we start from Nash equilibrium strategies, make the players deviate from them, and start increasing the available information. Furthermore, in [9] the authors are interested in the effects of information on the outcome of the game (which player wins), while we are interested in the deviations of the players over their time optimal strategies and the necessary information to detect such deviations to obtain optimality.

III. System Model



(a) Realistic space



(b) Reduced space

Fig. 1. System models

To simplify the problem, the game is modeled in a coordinate system that is fixed to the DDR (see Fig.1(b)), called in [3] the *reduced space*. In the reduced space all the orientations are measured with respect to the positive y-axis (DDR's heading). We denote the state of the system as $\mathbf{x}(t) = (x, y) \in \mathbb{R}^2$.The model of the kinematics in the reduced coordinate system is the following (see [1] for details).

$$\begin{aligned} \dot{x} &= \left(\frac{u_2 - u_1}{2b}\right) y + v_1 \sin v_2 \\ \dot{y} &= -\left(\frac{u_2 - u_1}{2b}\right) x - \left(\frac{u_1 + u_2}{2}\right) + v_1 \cos v_2 \end{aligned} \quad (1)$$

where $u_1, u_2 \in [-V_p^{\max}, V_p^{\max}]$ are the DDR's controls, and they correspond to the angular velocities of its wheels, u_1 is the angular velocity of the left wheel and u_2 of the right wheel. $v_1 \in [0, V_e^{\max}]$ and $v_2 \in [0, 2\pi)$ are the evader's controls associated to its speed and motion direction, respectively, in the reduced coordinate system. This set of equations is expressed in the form $\dot{\mathbf{x}} = f(\mathbf{x}, u, v)$, where $u = (u_1, u_2) \in \widehat{U} = [-V_p^{\max}, V_p^{\max}] \times [-V_p^{\max}, V_p^{\max}]$ and $v = (v_1, v_2) \in \widehat{V} = [0, V_e^{\max}] \times [0, 2\pi)$.

Inequality (2) gives the maximum rate of rotation $\omega^{\max}$ for the pursuer, given a specified linear speed ν [1], [5].

$$\begin{aligned} \omega &= \frac{u_2 - u_1}{2b} \\ \nu &= \frac{u_1 + u_2}{2} \\ |\omega^{\max}| &\leq \frac{1}{b}(V_p^{\max} - |\nu|) \end{aligned} \quad (2)$$

where ν is the DDR's translation speed and ω its angular speed.

The following definitions are used in the rest of the paper: $\rho_v = V_e^{\max}/V_p^{\max}$ is the ratio between the maximum translational speed of both players, and $\rho_d = b/l$ is the ratio of the distance between the center of the robot and the wheel location b and the capture distance l. Note that $l \geq b$, otherwise the capture distance would be located inside the robot.

IV. State Space Partition for Motion Policy

In this section a partition of the state space into mutually disjoint regions is presented. This partition was found in [1] using Isaacs' methodology [3], which combines the theory of optimal control and differential games. To make this paper self-contained, we include an Appendix with three lemmas obtained in [1], which are used in this work.

Fig. 2(a) shows a graphical representation of the regions integrating the partition of the first quadrant of the reduced space. The frontiers between regions are called singular surfaces [3]. In this partition, there are 4 singular surfaces [1]: universal surface (US, black bold line), transition surface (TS, red curve), the barrier surface (BS, magenta straight line) and dispersal surface (DS, orange line). If the pursuer applies its time-optimal motion policy the barrier (BS) cannot be crossed by the evader. The answer to the capture-escape question relies on whether or not the barrier divides the reduced space into two parts. Suppose the barrier separates it into two parts. If x is in the outer side then the DDR cannot force the capture. If the barrier fails to separate the playing space (as in Fig. 2(a)), then capture can always be attained by the DDR. [1] along with [2] provide an analysis regarding the BS for our problem yielding the next remark:

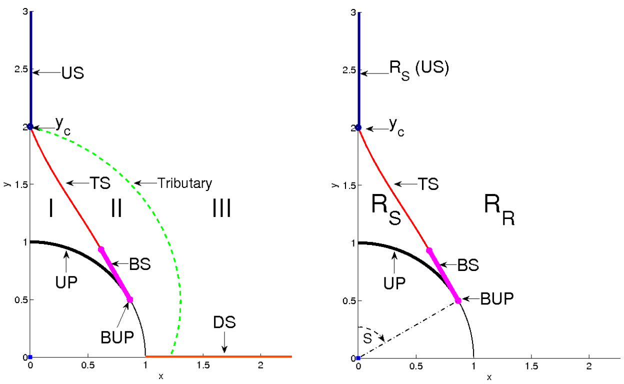
Remark 1: If the barrier does not separate the playing space for a given V_p^{max} and a given V_e^{max}, then the pursuer guarantees capture regardless of the strategy followed by the evader.

The universal surface (US) has the property that whenever the evader is located at it, the time-optimal motion policy for the pursuer is to move in a straight line to capture the evader. The limit of the US is at $y_c = l/\rho_v$ (see Fig. 2(a)).

The transition surface (TS) is the place where a control variable abruptly changes its value. In contradistinction with the US and the BS, the TS is not a trajectory traveled by the system in the reduced space. In the first quadrant, the TS represents the locus of points where the DDR switches one of its controls, in particular from Lemma 4 in Appendix I, we found that u_2^* switches from the value $V_p^{\max}$ to $-V_p^{\max}$. The expression defining the control u_2^* at the moment of the switch characterizes the conditions that must satisfy the points (x, y) in the reduced space.

A dispersal surface (DS) is defined in [3] as the locus of initial conditions along which the optimal strategy of one player or the optimal strategies of both players are not unique. At the DS, the choice of the control of one player must correspond to the choice of the control of the other player. Therefore, a solution will be to employ an instantaneous mixed strategy [3], which means the randomizing of a player's decision in accordance with some probabilistic law until the system is no longer on the DS.

The partition also contains the terminal surface and the usable part (UP). The terminal surface is the set of points that represents an opportunity for the DDR to capture the evader [3]. In this game it is a circle of radius l. The usable part (UP, black bold arc in Fig. 2(a)) is the portion of the space where the pursuer guarantees capture of the evader regardless of the choice of controls by the evader [3]. The boundary of the usable part is the point BUP shown in Fig. 2(a). In [1], the angle s denotes the angle measured from the positive y-axis to a point in the usable part, and $S = \cos^{-1}(\rho_v)$ denotes a bound in s corresponding to the boundary of the usable part (BUP). A more detailed description of each singular surface in the partition is presented in [1].



(a) Partition

(b) Partition for pursuer's feedback motion policy

Fig. 2. Partition of the first quadrant

In the interior of each region, the pursuer always applies its feedback-based time-optimal motion based on the evader's location over the reduced space. This policy for the first quadrant is summarized in Table I. In the remaining quadrants the pursuer time-optimal motion policy is analogous.

Evader in the reduced space	u_1, u_2
US	$u_1 = +V_p^{\max}$, $u_2 = +V_p^{\max}$
I	$u_1 = +V_p^{\max}$, $u_2 = +V_p^{\max}$
II	$u_1 = +V_p^{\max}$, $u_2 = -V_p^{\max}$
III	$u_1 = +V_p^{\max}$, $u_2 = -V_p^{\max}$
DS	Randomized strategy

TABLE I

Pursuer's feedback-based time-optimal motion policy in quadrant 1.

If the evader is located in Region I then the DDR moves in a straight line in the realistic space to capture the evader. Region II corresponds to configurations in the realistic space where the DDR initially rotates in place, but it is not necessary to align completely the DDR's heading with the segment joining the positions of both players in order to capture the evader. Region III in the reduced space corresponds to configurations in the realistic space where the DDR also rotates in place until it aligns its heading with the segment joining the players' position. The frontier between Region II and Region III is established by the tributary trajectory[1] (green dashed line) shown in Fig. 2(a).

[1] A tributary trajectory is an optimal trajectory of the system in the reduced space that reach the (US).

From Table I, we see that the US and Region I have associated the same optimal controls, and the same happens with regions II and III. Therefore, the partition shown in Fig. 2(a) might be simplified to one in which the US and Region I are merged and Region II and Region III are merged too. Hence, let R_S= US $\cup$ Region I and R_R = Region II $\cup$ Region III $\cup$ DS. Refer to Fig. 2(b).

V. Influence of the Available Information on the Motion Policy

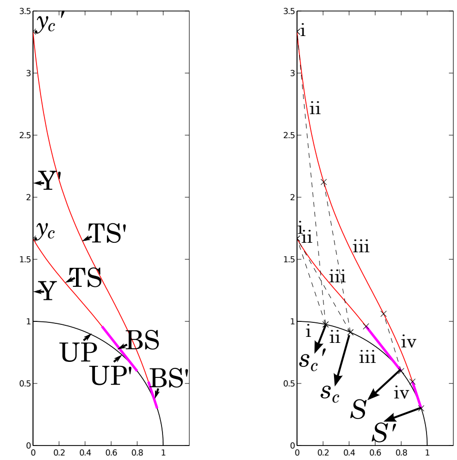
In this section we analyse the scenario in which the evader deviates from its optimal controls obtained in [1]. In particular we consider the case where the evader deviates from its optimal speed and the effects of this deviation over the pursuer strategy. As mentioned before, the motion strategy shown in [1] is in Nash equilibrium, namely, any unilateral deviation of a player from the optimal strategies does not provide it a benefit on its payoff. Nevertheless, Nash equilibrium does not elaborate on the existence of a new strategy that improves the payoff for a player that takes advantage over the deviation of other player on its optimal policy; neither it tells which extra information is needed in order to apply the new strategy if it exists. In this section through Theorem 1 we show that for the referred pursuit-evasion problem, there are cases where such new strategy for the pursuer exists when the evader deviates from its optimal speed, and that the extra information that that strategy uses–apart from the instantaneous evader's location (x, y) in the reduced space–, is the instantaneous evader's speed. The new motion strategy or policy for the pursuer is given in Definition 1. We also introduce Lemma 1 that will be constantly used along this section. Lemma 1 says that for any two partitions P and P' calculated as in [1], but P considering a maximum evader's speed V_e^{max}, and P' considering it as $V_e^{max'}$, with $V_e^{max'} < V_e^{max}$, then R_S is fully contained in R'_S.

Definition 1: To *apply the controls dictated by the instantaneous partition corresponding to* V_e, is to apply the controls dictated by a partition constructed as in [1] considering V_e as the maximum evader speed, where V_e is the current evader's speed.

Lemma 1: Let $V_e^{max'} < V_e^{max}$, and P' and P be state partitions of the reduced space built as in [1], using $V_e^{max'}$ and V_e^{max} as the respective maximum speeds. Then, $R_S \subset R'_S$, where R_S corresponds to P, and R'_S to P'.

Proof: In this proof we focus on the first quadrant of the plane but the obtained results can be easily extended to the other three quadrants. In the first quadrant the interior of region R_S (similarly for R'_S) is delimited by the usable part UP of the terminal surface, the barrier BS, the transition surface TS, and the portion of the y-axis, name it Y, going from the TS (point y_c) to the UP. What we proceed to do is to show that the boundaries of R_S are contained within the respective boundaries of R'_S or within R'_S itself.

a) Both UP and UP$'$ in the first quadrant run from an angle $s = 0$ to the angles $\cos S = {}^{V_e^{max}}/_{V_p^{max}}$ and $\cos S' = {}^{V_e^{max'}}/_{V_p^{max}}$, respectively (see Fig. 3(a)). Since $V_e^{max'} < V_e^{max}$, then $\cos S' < \cos S$ (this is, $S < S'$), hence UP $\subset$ UP'.



(a) Partition P and P' (b) s angle intervals of Lemma 1 proof

Fig. 3.

b) Y and Y$'$ (Y relates to R_S and Y$'$ to R'_S) run from UP to the points $y_c = l({}^{V_p^{max}}/_{V_e^{max}})$ and $y_c{}' = l({}^{V_p^{max}}/_{V_e^{max'}})$, respectively (see Fig. 3(a)). Since $V_e^{max'} < V_e^{max}$, then $y_c < y_c{}'$, hence Y $\subset$ Y$'$.

c) TS and TS$'$ are delimited in the first quadrant by points y_c and $y_c{}'$, and the top endpoints of the respective barriers BS and BS$'$. The coordinates of the points composing TS and TS$'$ can be parameterized making use of angle s. Making use of s we define four intervals, which we will use to show that TS is bellow TS$'$ withing R'_S. The four intervals are built setting s to 0, to angle $s_c{}' = \tan^{-1}(\rho_d({}^{V_e^{max'}}/_{V_p^{max}}))$, to angle $s_c = \tan^{-1}(\rho_d({}^{V_e^{max}}/_{V_p^{max}}))$, to angle $S = \cos^{-1}({}^{V_e^{max}}/_{V_p^{max}})$ and to angle $S' = \cos^{-1}({}^{V_e^{max'}}/_{V_p^{max}})$ (Lemma 5), yielding the following analysis:

   i) $s \in [0, s_c{}']$ (see Fig. 3(b)) In this interval for each value of s, for TS the optimal trajectories of the system do reach the y-axis at $y_c = l({}^{V_p^{max}}/_{V_e^{max}})$ [1], and for TS$'$ the optimal trajectories reach $y_c{}' = l({}^{V_p^{max}}/_{V_e^{max'}})$. As $y_c < y_c{}'$, it is evident that for this s interval TS is below TS$'$ and within R'_S.

   ii) $s \in (s_c{}', s_c]$ (see Fig. 3(b)) Regarding TS, in this interval each value of s maps TS to y_c, as the optimal trajectories of the system in retro-time, starting from the UP, do reach the y-axis at y_c since the time τ_c that takes the system to reach the y-axis is smaller than the time τ_s at which the DDR switches controls [1]. Eq. (3) gives us the y-coordinate, that is $y(s)$, if for the s values in the present interval we would have extended the optimal trajectory beyond the y-axis intersection up to time τ_s, meaning $y_c < y(s)$. Eq. (4) gives the y-coordinate, that is $y'(s)$, for TS$'$ in the present interval. By inspecting Eq. (3) and

Eq. (4), it can be seen that $y(s) < y'(s)$, and since $y_c < y(s)$, then $y_c < y'(s)$. Hence, for this s interval TS is below TS$'$ and within R'_S.

$$y(s) = b\cot s + l\cos s - b\frac{V_e^{max}}{V_p^{max}}\frac{\cos^2 s}{\sin s} \quad (3)$$

$$y'(s) = b\cot s + l\cos s - b\frac{V_e^{max'}}{V_p^{max}}\frac{\cos^2 s}{\sin s} \quad (4)$$

iii) $s \in (s_c, S]$ (see Fig. 3(b)) Inspecting Eqs. (3), (5), (4) and (6), we have that TS$'$ can be obtained by translating to the right and up every point in TS for every value of s in this interval, therefore, if in this interval TS and TS$'$ monotonically decrease on s, then for this s interval TS is below TS$'$ and within R'_S. This can be easily proven by calculating the derivative of Eq. (3) (or Eq. (4)) with respect to s and verifying that the resulting equation is negative in the given interval. As a result, for this s interval TS is below TS$'$ and within R'_S.

$$x(s) = l\sin s - b\frac{V_e^{max}}{V_p^{max}}\cos s \quad (5)$$

$$x'(s) = l\sin s - b\frac{V_e^{max'}}{V_p^{max}}\cos s \quad (6)$$

iv) $s \in (S, S']$ (see Fig. 3(b)) Inspecting Eqs. (3), (5), (4) and (6), we know that $(x(S), y(S))$ is to the left and below $(x'(S), y'(S))$, but since R_S ends at S but R'_S still continues up to S', R_S no longer exist for this interval and no comparison against R'_S can be done. Therefore, the three past cases are sufficient to prove that TS is below TS$'$ and within R'_S.

d) The BS runs from the point in the terminal surface at angle S towards the point $(x(S), y(S))$ obtained evaluating Eqs. (3) and (5) at angle S. From the analysis regarding the transition surfaces, we know that such point $(x(S), y(S))$ is bellow TS$'$, and since $S < S'$ (R'_S still extends from S to S'), then BS is within R'_S.

■

Remark 2: Lemma 1 splits the playing space in the first quadrant, into three regions: R_S, $R'_S \setminus R_S$ and R'_R.

Theorem 1: Call π to the pursuer motion strategy dictated by the state space partition calculated as in [1], that is, considering pursuer and evader maximum speeds, V_p^{max} and V_e^{max}. Now, consider that the pursuer has access to the instantaneous evader speed V_e, with $0 < V_e \leq V_e^{max}$, and that as its motion strategy the pursuer applies the controls dictated by the instantaneous partition corresponding to V_e. Refer to this strategy as σ. There are scenarios where the strategy σ yields a smaller capture time than applying π.

Proof: Consider the scenario of an evader with a maximum speed V_e^{max}, but that it will be moving with a constant speed V_e, with $0 < V_e < V_e^{max}$. Let us call P to the state space partition that encodes strategy π, calculated considering V_e^{max} as the maximum evader speed. Let us refer as P' to the state partition constructed as in [1] but considering V_e as the maximum evader speed.

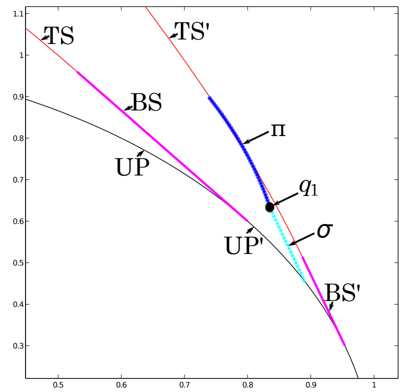


Fig. 4. Example of Theorem 1

Assume that the game starts with the evader within $R'_S \setminus R_S$ (R_S refers to P and R'_S to P') at a point q_1, see Fig. 4. For this configuration, strategy π, making use of P, indicates the pursuer to rotate until R_S (delimited in Fig. 4 by BS and TS) is reached and then to apply a straight motion until the capture is achieved. As the evader will be moving with constant speed V_e, strategy σ will only make use of partition P'; for such initial configuration P' tells the pursuer to apply a straight line motion until capture is achieved. Now, consider the parameters $l = 1$, $b = 0.75$, $V_p^{max} = 1$, $V_e^{max} = 0.6$, $V_e = 0.3$ and $q_1 = (0.8363, 0.6261)$. For these parameter values, the pursuer while applying σ captures the evader in 0.2 time units (cyan–light grey–trajectory in Fig. 4, starting in q_1 moving towards UP$'$), and while applying π the pursuer does not even manage to take the evader to R_S in the same 0.2 time units (blue–dark grey–trajectory in Fig. 4, starting in q_1 moving towards R_S). For this parameters, the strategy σ yields a smaller capture time than applying π. The result follows. ■

Corollary 1: Following a similar reasoning as in Theorem 1 proof, a family of innumerable examples can be built where the strategy σ yields a smaller capture time than applying π.

Nonetheless, there are also cases in which despite the availability of new information, namely the instantaneous evader speed, the pursuer cannot improve its payoff, as it must apply the worst case strategy (the one from [1] that is in Nash equilibrium), otherwise the capture of the evader is not longer guaranteed. Lemma 2 presents a family of pathological examples, such that when the pursuer applies the controls dictated by the instantaneous partition corresponding to the evader's instantaneous speed V_e, then the evader capture is not guaranteed even when the conditions mentioned in Remark 1 were met, meaning, the capture was possible applying the strategy given by the partition corresponding to V_e^{max}. Making use of Lemma 2 and other arguments we introduce Theorem 2, which says that there are states in the reduced space where knowing V_e is not of relevance, hence, it can be discarded.

Lemma 2: Consider a pursuer that has access to the evader's instantaneous speed V_e, with $0 < V_e \leq V_e^{max}$, and that such pursuer applies the controls dictated by the instantaneous partition corresponding to V_e. Such pursuer's strategy can make the evader move in a cycle never reaching the UP, preventing the pursuer from capturing the evader, even if in the instantaneous partitions the barrier does not separate the playing space.

Proof: Assume that the evader will be moving interchanging between two speeds, one being V_e^{max}, and the other V_e, with $0 < V_e < V_e^{max}$. Then name P the partition built considering V_e^{max} as the maximum evader speed, and P' the partition built considering V_e as the maximum evader speed. Also assume that the barrier does not separate the playing space in neither P nor P', because if that does happen then from [1] we already know that the capture could be avoided by the evader. Finally, assume that the evader starts within $R'_S \setminus R_S$ (point q_1 in Fig. 5), that it starts moving with speed V_e^{max} and that it will be moving in the direction dictated by the instantaneous partition corresponding to the instantaneous evader speed. For this initial system configuration, as the evader is moving with speed V_e^{max}, then the pursuer strategy tells it to move with a rotation on site, moving the system over the curve $C2$ shown in Fig. 5. Then consider that the evader arriving to point q_2 it decides to move at speed V_e, then the pursuer strategy will tell it to move in a straight line, moving the system along the curve $C1$ in Fig. 5. If the evader decides to move again at speed V_e^{max} while arriving to point q_1 then the system would move again towards point q_2. This evader's speed interchange could be indefinitely repeated by the evader at points q_1 and q_2, producing the system to keep indefinitely oscillating avoiding the evaders capture. The result follows.[2] ■

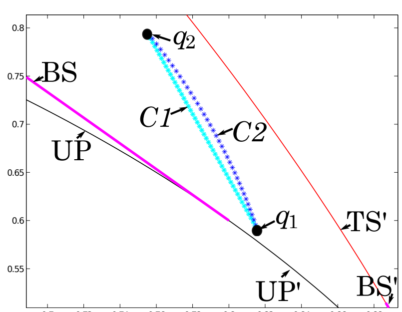


Fig. 5. Example of Lemma 2

Theorem 2: Consider a pursuer that has access to the evader's instantaneous speed V_e, with $0 < V_e \leq V_e^{max}$, and that such pursuer applies the controls dictated by the instantaneous partition corresponding to V_e. There are states in the reduced space, where such pursuer's strategy has no benefit apart from the pursuer applying the partition corresponding to V_e^{max}, hence, knowing V_e is not of relevance on those states and it can be discarded.

[2]Fig. 5 was produced with a simulation, hence, parameters that produce the described behaviour do exist.

Proof: Without lost of generality, suppose that the evader is currently traveling with a speed $Ve < V_e^{max}$, to which the pursuer has access to. Next, we proceed to give sets of evader's positions where knowing V_e does not improve the payoff for a pursuer that applies the partition corresponding to V_e^{max}.

By Lemma 1, in the partition corresponding to V_e^{max}, let us call it P, region R_S is delimited by any other region R'_S in a partition P' tied to a fixed speed $V_e < V_e^{max}$. If over the reduced space the evader is within region R_S, for both partitions P and (any) P' the dictated pursuer's control is the same, hence, the pursuer achieves capture in both partitions travelling in a straight line. This can be seen as the pursuer could discard the instantaneous speed and apply the controls dictated by partition P.

If the evader is within $R'_S \setminus R_S$, partition P dictates the pursuer to travel in a straight line and P' that the pursuer rotates on site. Furthermore, in $R'_S \setminus R_S$ is the place in the reduced space where the pathological example described in Lemma 2 exists. At the states where the pathological example might take place the pursuer does not know if the evader will have such behaviour by just knowing the instantaneous evader's speed. This indicates that in those states the pursuer should opt for the strategy dictated by P, as such partition achieves capture for each evader's speed V_e bounded by V_e^{max} (Remark 1), and is the most restrictive partition (the partition with the smallest usable part, Lemma 1) as it is the only one that guarantees capture of the evader in the worst case (evader traveling at V_e^{max}) when it is outside region R_S (any other partition P' makes the pursuer move in straight line before time). In those states the pursuer should forget about the partition P' related to the current instantaneous speed and concentrate on partition P, that is, to discard the current instantaneous speed.

If the evader is in R'_R, the controls for the pursuer dictated by partitions P and P', are the same, which make the pursuer to rotate on site. As a consequence the instantaneous evader's speed is not useful and the pursuer can discard it. If the pursuer keeps rotating on site it eventually takes the system to $R'_S \setminus R_S$, which takes us the scenario analysed above, or to the universal surfaces where the partitions again agree that the pursuer must travel on a straight line with V_p^{max} and the evader's speed can be discarded.

By the past analysis we point out several states in the reduced space (indeed, states over R_S, $R'_S \setminus R_S$, and R'_R), where a pursuer that applies the controls dictated by the instantaneous partition corresponding to V_e has no benefit apart from applying the partition corresponding to V_e^{max}, hence, knowing V_e is not of relevance on those states and it can be discarded. Note, that the same analysis can be applied for any $V_e < V_e^{max}$. The result follows. ■

The importance behind Theorem 2 is that a gain in information does not necessarily translate into a gain in the game outcome. Indeed, in Theorem's 2 proof, we have identified two kind of scenarios where this happens. The first one, where the deviations on speed are irrelevant (region R_S in the proof) and therefore knowing the instant value of V_e is irrelevant; and the second one, where the deviations are relevant but the extra information is not enough to motivate the pursuer to leave its worst case strategy (Lemma 2).

In the later scenario, knowing the instantaneous speed V_e does not foresee the possible speed deviations that the evader can follow. This means that the lack of certainty on the future speed will make the pursuer to stick to the worst case analysis. This suggest that a search for the useful information is needed, that is *information regarding to relevant deviations* (this might depend on the state of the system), and *this information must mitigate uncertainty in such a way that the pursuer is not obligated to apply a worst case analysis*. (For instance, some other information apart from the instantaneous speed would be needed in order for the pursuer to differentiate when it is in the case described in Theorem 1 from the one in Lemma 2.)

VI. CONCLUSIONS AND FUTURE WORK

In this paper, we have analysed the scenario in which the players deviate from their optimal controls. We have shown that when the evader deviates from its optimal speed there are cases where there exists a new pursuer motion strategy that reduces the time to capture the evader; these cases require more information about the evader's state, which was the instantaneous evader's speed. Nevertheless, we have exhibited cases in which despite the availability of new information, the pursuer must stick to the worst case strategy, otherwise it cannot capture the evader.

Based on the presented analysis, we have introduced the necessity of finding the required information to identify relevant deviations from the optimal strategies of the players, and such information must mitigate uncertainty in such a way that the players are not obligated to apply a worst case analysis.

A hypothesis for further research is that in order to achieve time optimality for any situation in the execution stage, the necessary information to foresee future deviations must be found, and such information might be all the necessary components to reconstruct the full evader trajectory in advance in the planning stage, otherwise in the planning stage a worst case analysis as the Nash equilibrium is needed which might not produce time-optimal trajectories if the evader deviates from its optimal policy.

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APPENDIX I
PREVIOUS SUPPORTING RESULTS

In this appendix, we present three lemmas obtained in [1], which are used in this work. For the proofs please refer to [1]. In [1] $\tau = t_f - t$ denotes the retro-time, in which t_f is the termination time of the game.

Lemma 3: The barrier consists of a straight line segment, and it intersects the y-axis in the first quadrant if $\rho_v \geq |\tan S|/\rho_d$ where $S = \cos^{-1}(\rho_v)$ is the angle at the BUP.

This Lemma implies that for $S = \cos^{-1}(V_e/V_p)$ then $\tau = (b\cos S)/(V_p \sin S)$. See [1] for more details.

Lemma 4: The DDR switches controls and it starts a rotation in place in the realistic space, at $\tau_s = |\frac{b\cos s}{V_p^{\max}\sin s}|$. If $s \in [0, \pi]$, u_2^* switches first, otherwise u_1^* does.

Lemma 5: The straight lines trajectories that have an orientation $s \in (\tan^{-1}(\rho_v\rho_d), \cos^{-1}(\rho_v)]$ in the UP of the first quadrant terminate when the DDR switches controls.