Translation and rotation in the plane

CIMAT

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Material taken form the book: J. C. Latombe, Robot motion planning.

The setup

The working space is $W = \mathbb{R}^2$.

The robot A is line segment of length d, with endpoints P and Q.

The point P is located at (x, y), the orientation is taken from an angle $\theta \in [0, 2\pi)$.

Thus the configuration space C of A is $\mathbb{R}^2 \times S^1$, each configuration is (x, y, θ).

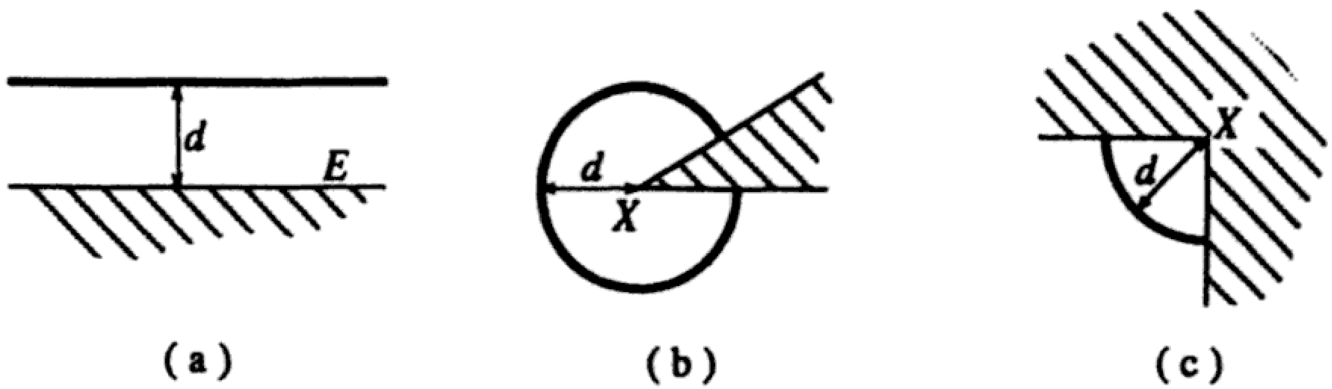
The union of all obstacles form a polygonal region B.

CB is a three dimensional region in $\mathbb{R}^2 \times [0, 2\pi)$.

Critical curves

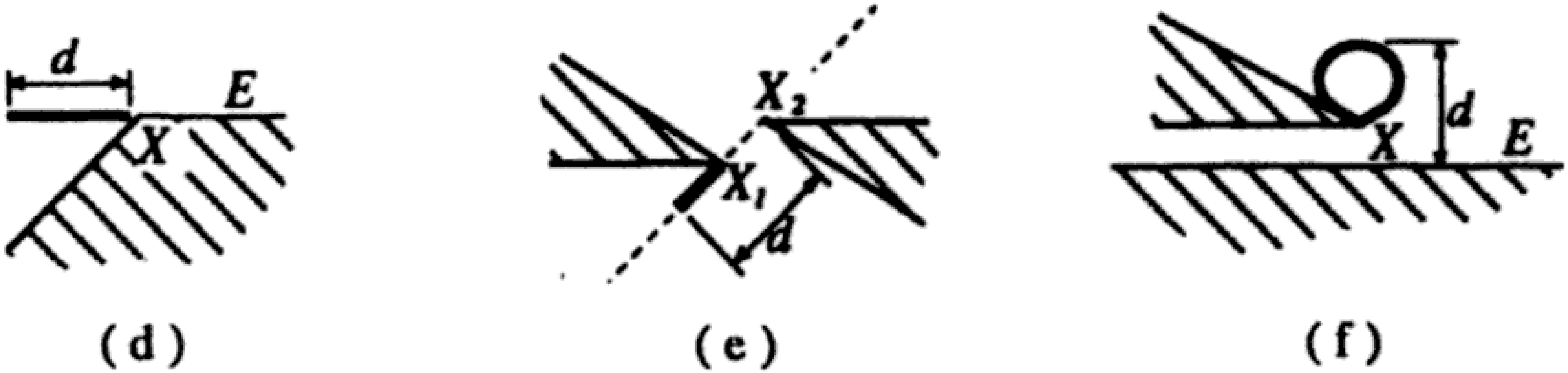
These curves can be expresed with an algebraic formula.

Obstacle edges are said to be critical curves of **type 0**.



a) The line segment at distance d from an obstacle edge E. This line has the same length as E. E is a critical curve of **type 1**.

b) and c) The arc of radius d, centered at the obstacle vertex X and bounded the two lines that form the vertex. X is a critical curve of **type 2**.



d) Let E be an obstacle edge and X a convex vertex at one endpoint of E. PQ is contained in the line containing E. The line traced as A slides and Q is in E is a critical curve of **type 3**.

e) X_1 and X_2 are two convecx obstacle vectices, A is bitangent to them. The line traced by P as A moves is a critical curve of **type 4**.

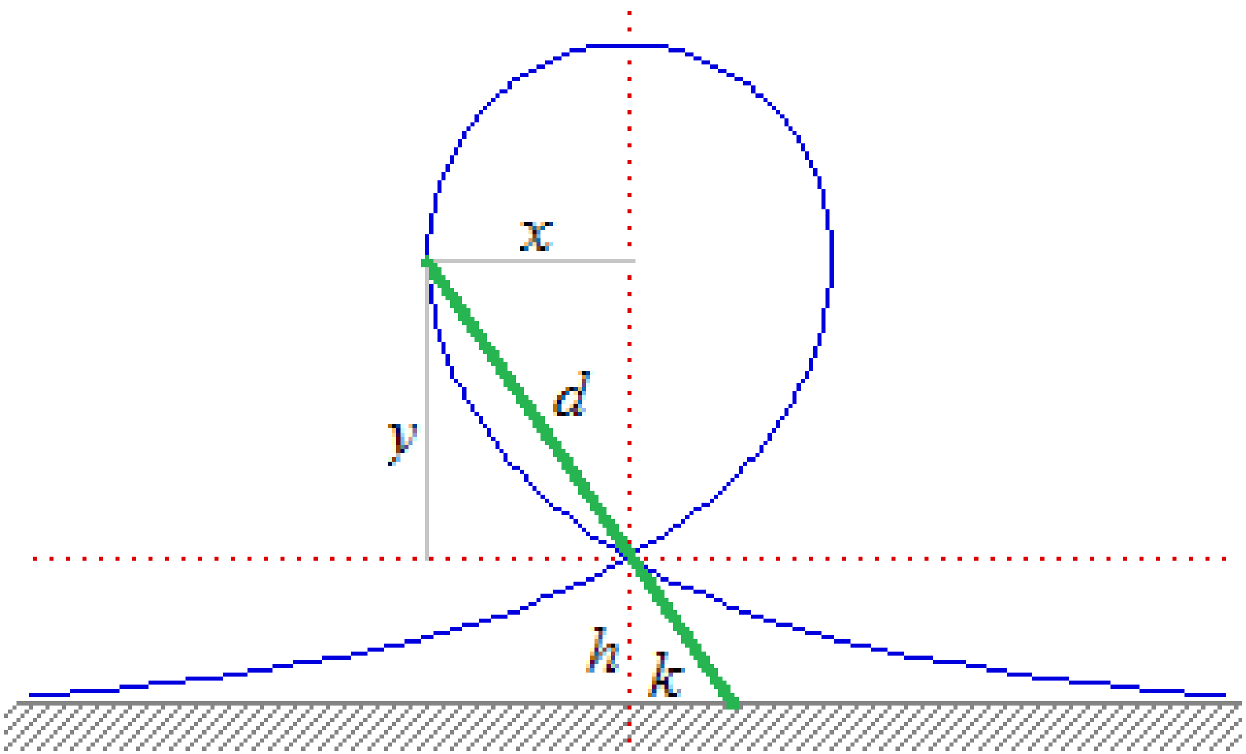
f) E is and obstacle edge, X is a convex obstacle vertex that is not and endpoint of E. The distance between E and X is h. If $h<d$, the curve traced by P, as A moves in a way such that it touches both E and X is a critical curve of **type 5**.

This curve is a conchoid of Nicodemes, with

$$d^2=(y+h)^2+(x+k)^2,$$

$\frac{y}{x}=\frac{h}{k}$, thus

$$x^2=y^2\left(\frac{d^2}{(x+h)^2}-1\right).$$



The set of critical curves is finite.

Every critical curve is a smooth curve algebraic curve of degree 1 (0, 1, 3, 4), 2 (2), or 4 (5).

The configuration space will be divided using this critical curves using a connectivity graph G.

G is an undirected graph, each node is a cell in C. Two nodes are connected if the cells that they represent are adjacents.

A position (x, y) is admisible if there exists at least one orientation θ such that

$$(x, y, \theta) \in C_{\text{free}}.$$

A non-critical region R is defined in base of a contact of A with obstacles, it is defined as

$$F(x, y) = \{\theta \mid (x, y, \theta) \in C_{\text{free}}\}.$$

If A is free for all θ, then

$$F(x, y) = [0, 2\pi),$$

else

$$F(x, y) = \text{a finite set of intervals}$$

For each maximally connected interval $(\theta_1, \theta_2) \subseteq F(x, y)$, let s_1 and s_2 the contacts by $A(x, y, \theta_1)$ and $A(x, y, \theta_2)$. The contacts could be on either a vertex X or an edge E.

Each interval has associated two contacts, one is a clockwise (at θ_c) the other is counterclockwise (at θ'_c).

Let $\sigma(x, y)$ be the set of all the pairs $[s(x, y, \theta_c), s(x, y, \theta'_c)]$, the interval $(\theta_c, \theta'_c) \subseteq F(x, y)$.

If $F(x, y) = [0, 2\pi)$, we write $\sigma(x, y) = \{[\Omega, \Omega]\}$.

Given a pair $[s_1, s_2] \neq [\Omega, \Omega]$, we denote an unique orientation $\lambda(x, y, s_1)$ such that $A(x, y, \lambda(x, y, s_1))$ touches the clockwise stop s_1.

Let R be a non-critical region.

A cell is defined as

$$cell(R, s_1, s_2) = \{(x, y, \theta) \mid (x, y) \in R \wedge \theta \in (\lambda(x, y, s_1), \lambda(x, y, s_2))\}.$$

Two cells $\kappa = cell(R, s_1, s_2)$ and $\kappa' = cell(R', s'_1, s'_2)$ are adjacent if and only if:

- The boundaries ∂R and $\partial R'$ of R and R' share a critical curve section β, and
- $\forall (x, y) \in \text{int}(\beta) \; : \; (\lambda(x, y, s_1), \lambda(x, y, s_2)) \cap (\lambda(x, y, s'_1), \lambda(x, y, s'_2)) \neq \emptyset$