Gap Navigation

CIMAT

Miguel Vargas

Material taken form the paper: B. Tovar, R. Murrieta-Cid, S. M. LaValle. Distance-Optimal Navigation in an Unknown Environment Without Sensing Distances. IEEE Transactions on Robotics, Vol. 23, No. 3. June 2007.

The idea: Do more with less

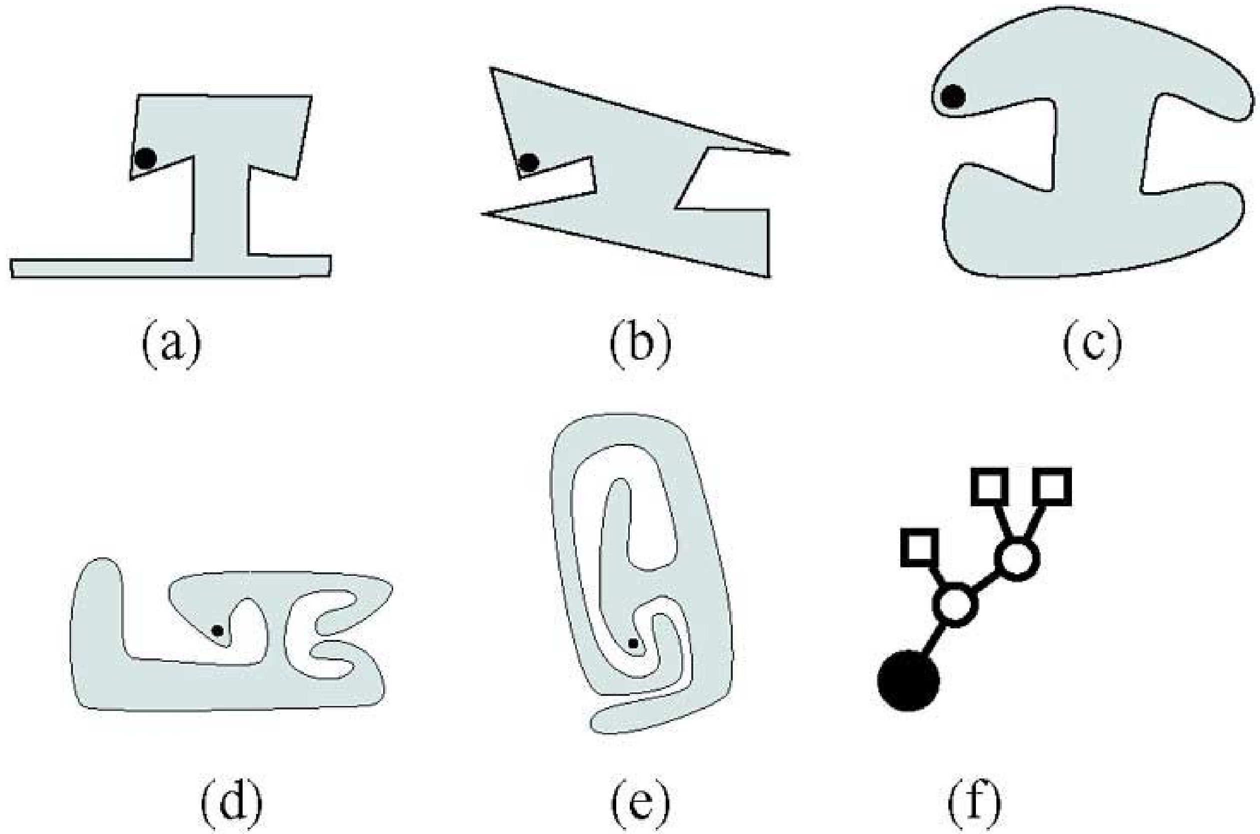
The setup:

- Unknown finite 2D environment, its free area is denoted with R
- A robot (a dot)
- The robot has an omnidirectional sensor that can identify discontinuities (gaps) on the boundary ∂R

The goals:

- Construct a complete navigation map
- Determine optimal paths, in euclidean sense. A ***path*** is defined as $\tau : s \rightarrow R,\ s \in [0, 1]$

The map

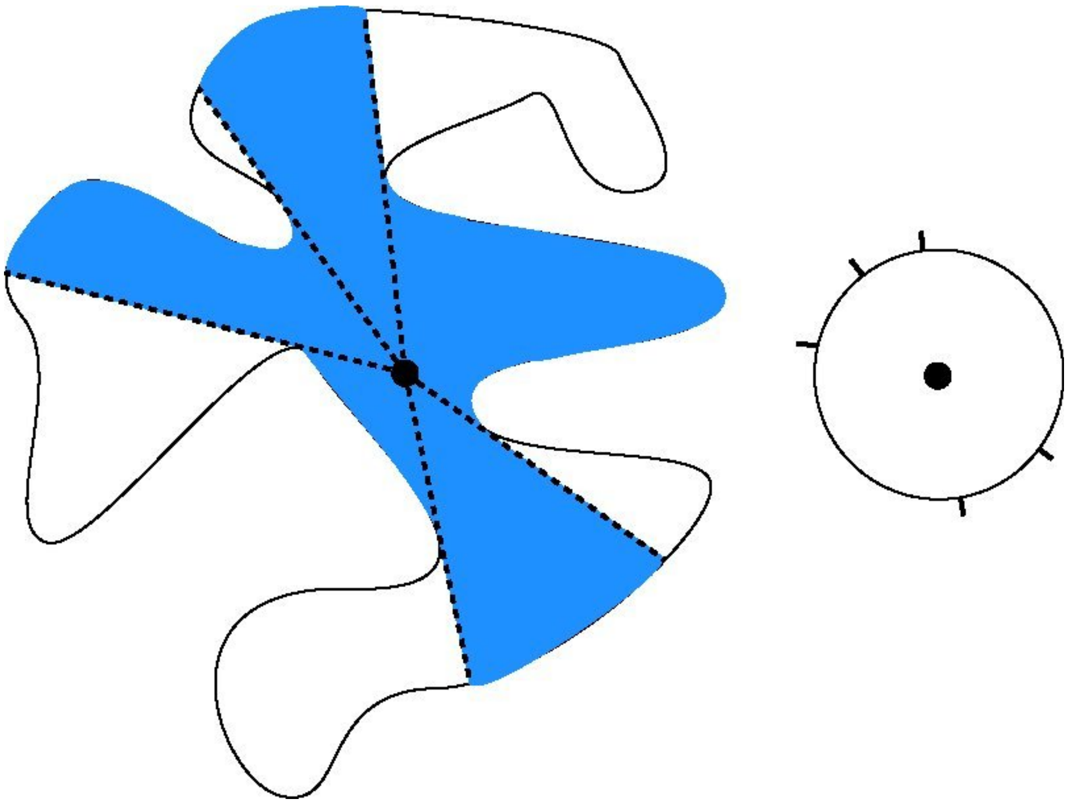


Environments (a), (b), (c), (d) and (e) are represented by the same graph (f)

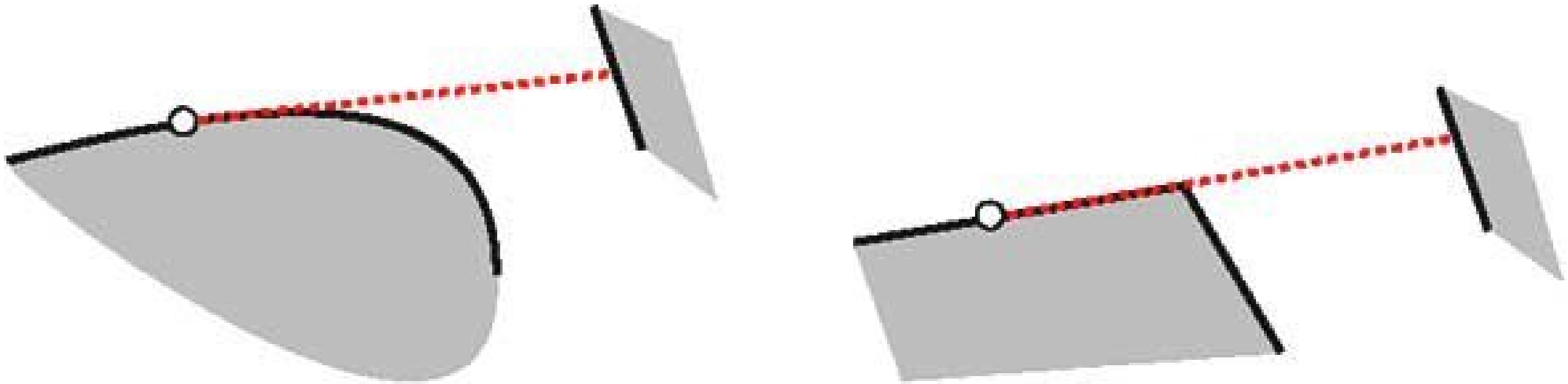
The representation of the map is a graph (a tree)

- It does not describe the exact shape of the environment
- The graph is constructed with the movements of the robot in the environment
- The graph can be used to set a path to translate the robot

The sensor



- This sensor detects and tracks discontinuities based on abrupt depth changes (***gaps***)
- The only information about the gaps are their sequence or order in a circle; $G(x)=[g_{1,}\ldots, g_k]$
- Several positions may have the same representation (small movements)
- As the robot moves gaps can change to a different combination
- The robot is able to label gaps, there is not confusion identifying them



(a) (b)

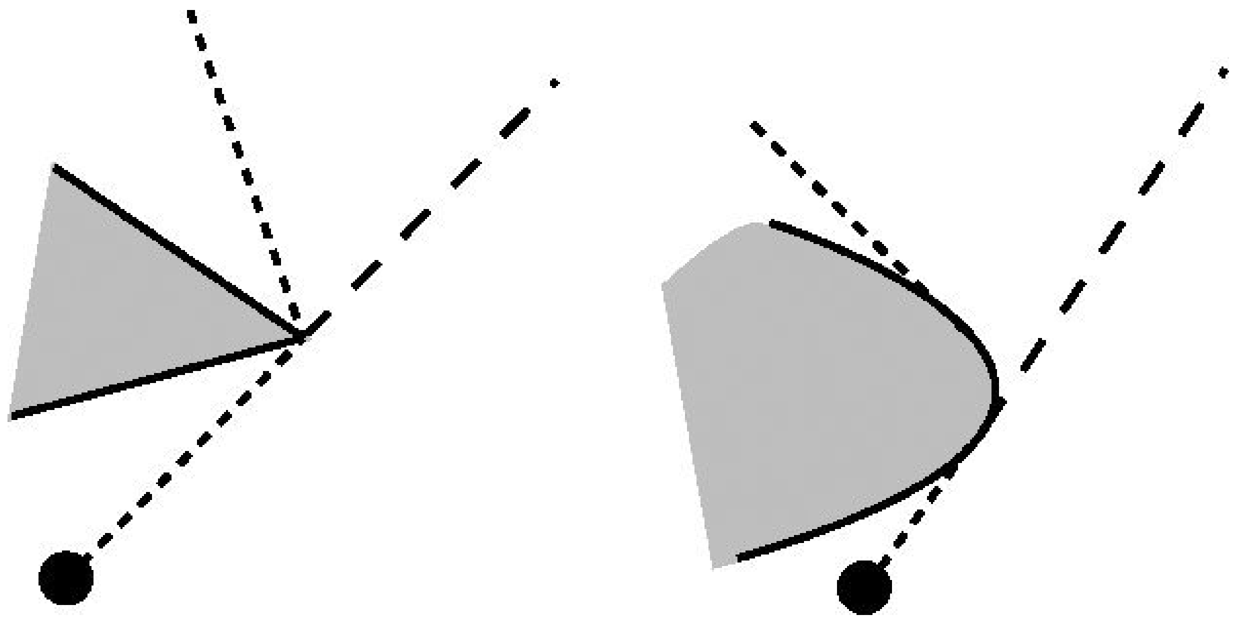
For a robot that moves along a boundary, a gap will appear if

(a) The boundary is curved smoothly and the ray touches it in a tangent point and part of the curve goes beyond the ray.

(b) The boundary is strait and the ray is tangent to it up to a certain point where the boundary changes direction beyond the ray.

Motion

There is not localization in terms of coordinates



Gap ***chasing*** $chase(g)$

- The robot rotates to align its direction to a certain gap and moves to it with unit speed
- The robot is able to avoid collisions
- If the gap is formed by a smooth boundary the robot follows it tangentially
- The chase terminates when when the gap disappear

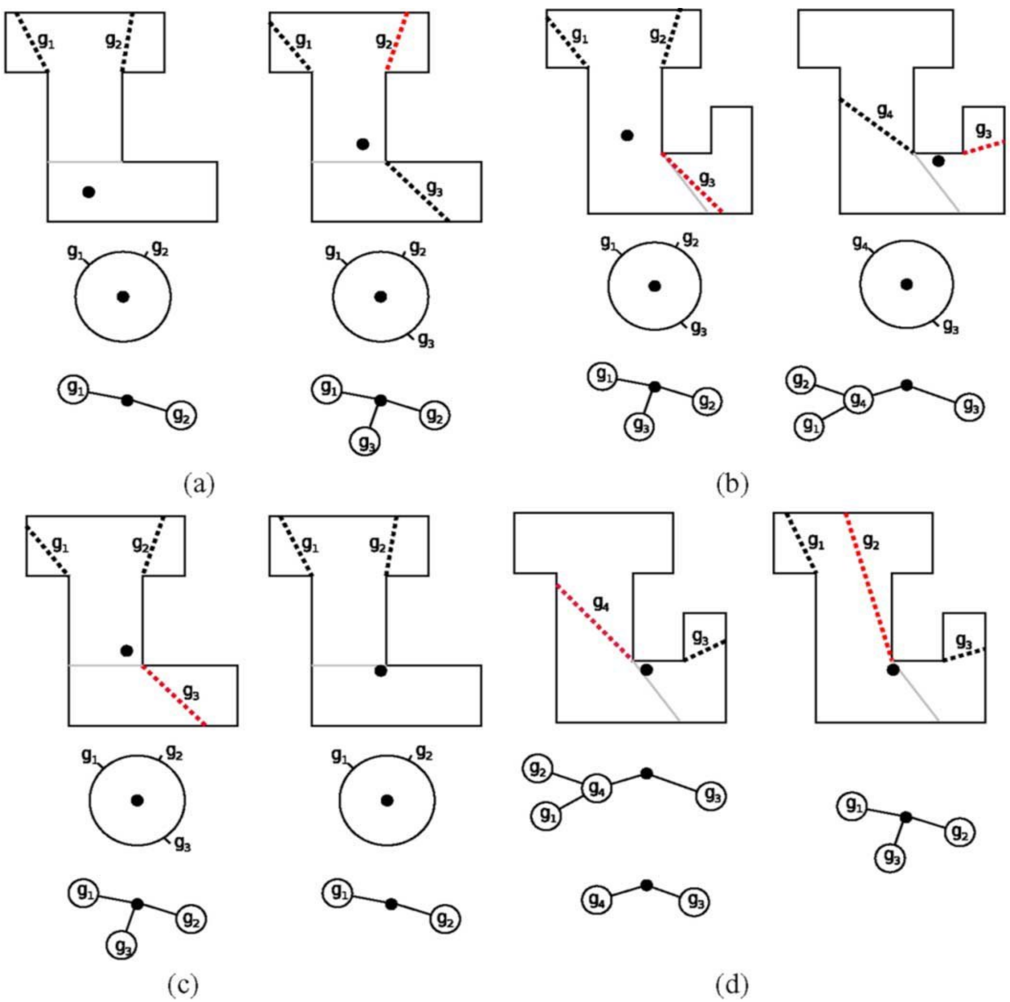
All motion strategies are based in finite sequences of this primitive

Gap navigation tree (GNT)

Initially it consists of a root vertex that is connected to a leave for every gap

The tree is constructed incrementally as the robot moves

There are four kinds of ***critical events*** that can happen, these define how the tree is build



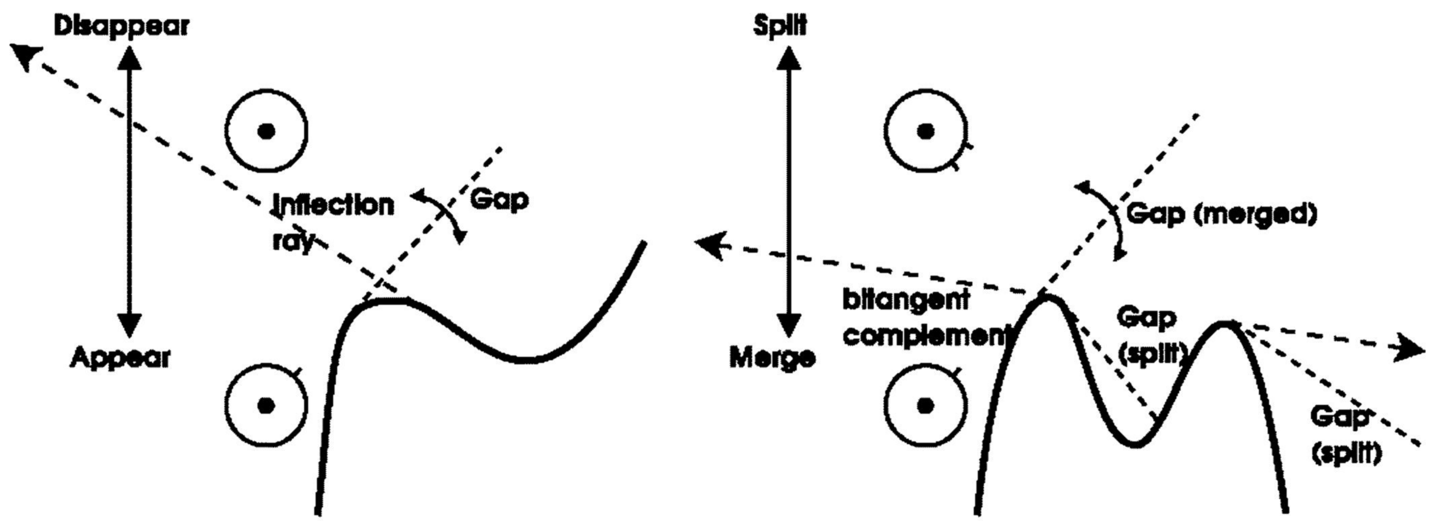
(a) A new gap appears: A vertex g is added as a child of the root (preserving the cycling order)

(b) Gaps g_1 and g_2 merge into g: Nodes g_1 and g_2 become children of a new node g, then (a)

(c) A gap g disappears: The leaf node g is removed

(d) Gap g splits into g_1 and g_2: If g is a node, g_1 and g_2 become new nodes, else they are already nodes of g. g_1 and g_2 are added as leaves of root

Geometric interpretation of critical events



(a) (b)

(a) Generalized inflection
Gaps appear or disappear when the robot crosses inflection rays

(b) Generalized bi-tangents
Splits and merges of gaps occur when the robot crosses bi-tangent complements

Inflection rays and bi-tangent complements decompose R into cells of similar visibility

No line is tangent to more than two points of the boundary

Lemma 1

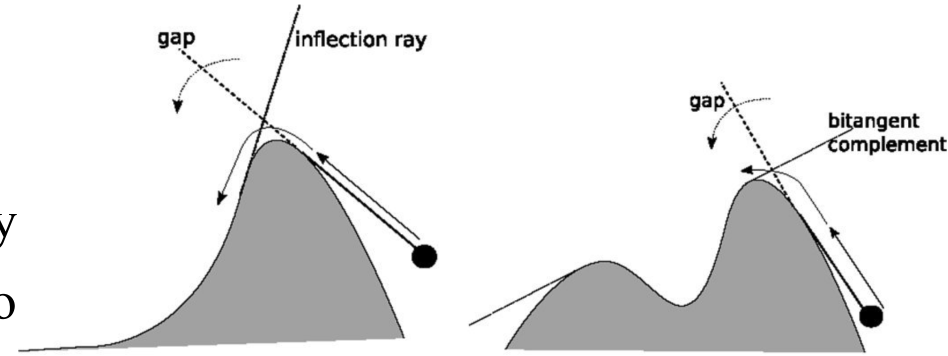
Let g_1 and g_2 be two gaps that merge into gap g_3. When g_3 splits, g_1 and g_2 appear at the same angular position in R at the time of the merge, independently from the robot motion.

Proof: Merges and splits occur when the robot crosses a bi-tangent complement of ∂R. Thus, g_1, g_2 and g_3 are aligned with the bi-tangent at the split or the merge. This is independent of where the bi-tangent complement is crossed. ■

“Lemma 1 implies that, when a gap splits, it can only split into gaps that merged before”

“The identification of the gaps is done purely by the order of the gaps before the corresponding merge and not by the “features” of the environment that produced them”

Lemma 2



(a) (b)

Termination of $chase(g)$ is guaranteed for any $g \in G(x)$ and any $x \in R$ and is caused by only two possible critical events: disappearance or splitting of g

Proof: The heading of the robot is always aligned with g, which forbids the robot to follow any cycle, or to move away from g. As the robot moves with unit speed towards the gap, the starting point of the gap slides on ∂R, towards the respective inflection ray or bi-tangent complement.

When the robot reaches ∂R, the position of the robot and the starting point of the gap coincide, and three cases should be considered: 1) the robot moves away from the inflection ray or bi-tangent complement; 2) the robot is stationary in ∂R; and 3) the robot moves towards the inflection ray or bi-tangent complement.

Cases 1 and 2 cannot occur, since the heading of the robot always points to the gap, and the robot moves tangentially on ∂R with unit speed. Thus, the remaining case always occurs, and the respective inflection ray or bi-tangent complement is eventually crossed. It is clear that the termination critical event cannot be an appearance, since the gap is already detected $g \in G(x)$.

The critical event cannot be a merge either, because the corresponding merging gap for the bi-tangent complement pair is not yet visible. ■

Complete Gap Navigation Tree

When is a GNT as complete as possible for a particular environment?

If any leaf vertex has the potential to split, then the GNT is incomplete because it could expand.

Some gaps split when approached using $chase(g)$ and others simply disappear. Let the gaps that disappear and their corresponding vertices in the GNT be called ***primitive***.

If all leaves of a GNT are primitive, then the GNT is said to be complete.

Lemma 3

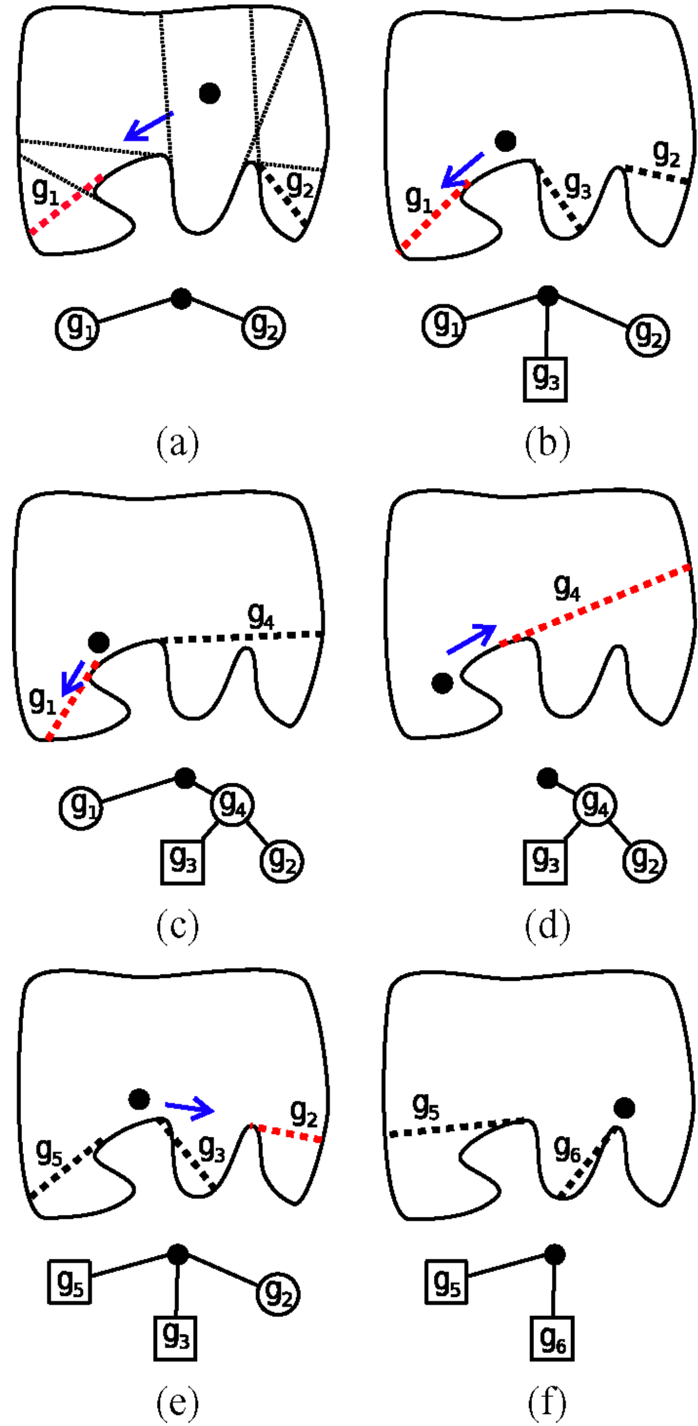
The procedure of iteratively chasing non-primitive leaves terminates with a resulting complete gap navigation tree.

Proof: Consider the path τ executed during the procedure. The key observation is that, any time that a new gap appears in $G(\tau(s))$, it must be primitive.

If the gap is chased, it cannot split. Therefore, the only gaps that contribute to the incompleteness of the GNT are ones that either appeared in $G(\tau(0))$ or were formed by a sequence of splits of these gaps.

Even though chasing a leaf may reveal new gaps via splitting, the number of primitive gaps for a given environment is finite because each corresponds to a inflection. There are finitely many inflections because ∂R is piecewise-analytic.

Each time that the procedure forces a gap to disappear, it is one step closer to having a complete GNT. Since the number of gaps is finite, the procedure must terminate with a complete GNT. ■



It is important to note that:
By lemma 3, the GNT is complete, but its representation could change as the robot moves in the environment.