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A Comparison of Two Methods for Spectral Analysis of Waves

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ABSTRACT
We consider the evolution of spectra of random waves over periods of three days. Two segmentation methods are used: Detection of Changes by Penalized Contrasts (DCPC) proposed and developed by Lavielle (1998, 1999) and Smooth Localized complex EXponentials (SLEX) proposed in Ombao et al. (2001). We compare the results obtained with both methods. In each case the intervals obtained are considered stationary and the corresponding spectra are obtained. For both sets of intervals the classical Fourier spectrum is obtained using the WAFO software. We compare some of the spectra obtained. We also apply both methods to the Hurricane Camille wave height data.

KEY WORDS: Spectral analysis; random waves; detection of changes by penalized contrasts; SLEX; stationarity periods.

INTRODUCTION
A simple approach to building long-term models of random waves is to assume that they are piecewise stationary random processes, i.e. that there are instants where the 'state' of the waves changes but in-between these change-points the waves are a stationary process. One advantage of this approach is that the classical spectral analysis can be used in each stationarity interval with the usual interpretation of the spectrum as the distribution of energy in a range of frequencies.
To implement this approach it is necessary to have ways of detecting changes in the state of the process, and since the spectra characterizes the covariance structure of a stationary process it is reasonable to look for methods based on changes of the spectra. In this work we compare two such methods: Detection of Changes by Penalized Contrasts (DCPC) Lavielle (1998, 1999), Lavielle and Ludeña (2000), and Smooth Localized complex EXponentials (SLEX) Ombao et al. (2002). Both methods have been implemented by their authors in Matlab and have been successfully used in other areas, particularly for the analysis of EEGs.
To compare their performance we considered three sets of data. The first two correspond to waves in a normal situation while the third set is the Hurricane Camille data, which corresponds to a highly non-stationary situation. In all cases we compare the stationarity intervals obtained by both methods and also study three characteristics of the spectra: total energy, maximum value and dominant frequency. We chart and compare the evolution of all three for both segmentation methods and for the three data sets.
In the next two sections we give brief descriptions of both methods, then we consider the two data sets corresponding to normal conditions and finally we study Hurricane Camilles's data.

THE DCPC METHOD
The problem of estimating the change-points of a sequence of a piece-wise stationary random process has received considerable attention in the literature (see, for example, Brodsky & Darkhovsky (1993), Basseville & Nikiforov (1993)). We describe here briefly a method proposed by M. Lavielle (1998, 1999) and studied in detail by Lavielle and Ludeña (2000).
Consider a sequence of real random variables $Y_1,\ldots,Y_n$ and assume that the distribution of the process depends on a parameter θ that changes abruptly at some unknown instants $(t_j,\ 1 \le j \le K)$, where K is also unknown. To estimate both K and the change-points $(t_j,\ 1 \le j \le K)$ a penalized contrast function of the form

$$J(\mathbf{t},\mathbf{y}) + \beta pen(\mathbf{t})$$

is used. The term $J(\mathbf{t},\mathbf{y})$ estimates the change points while the penalization term prevents the algorithm from overestimating the number of change-points. The latter only depends on the dimension $K(\mathbf{t})$ of the model and grows with K. The penalization parameter β adjusts the balance between the minimization of $J(\mathbf{t},\mathbf{y})$, which typically requires large values of K, and the minimization of $pen(\mathbf{t})$, which goes in the other direction.
The general principle proposed in the DCPC algorithm is the following: For any $1 \le k \le K$ let $U(Y_{t_{k-1}+1},\ \ldots,Y_{t_k};\theta)$ be a contrast function useful for estimating the unknown value of the parameter θ. The minimum contrast estimator $\hat{\theta}(Y_{t_{k-1}+1},\ldots,Y_{t_k})$ calculated on the k-th segment of **t** is defined as the solution of the following minimization problem:

$$U(Y_{t_{k-1}+1},\ldots,Y_{t_k};\hat{\theta}(Y_{t_{k-1}+1},\ldots,Y_{t_k})) \le U(Y_{t_{k-1}+1},\ldots,Y_{t_k};\theta),$$

for all $\theta \in \Theta$. For $1 \le k \le K$ let

$$C(Y_{t_{k-1}+1},\ldots,Y_{t_k}) = \frac{1}{n}U(Y_{t_{k-1}+1},\ldots,Y_{t_k};\hat{\theta}(Y_{t_{k-1}+1},\ldots,Y_{t_k}))$$

then the contrast function is defined as

$$J(t,y) = \sum_{k=1}^{K} C(y_{t_{k-1}+1}, \ldots, y_{t_k})$$

where $t_0=0$ and $t_K=n$.

Using this general principle, different contrast functions can be used according to the situation. In the case of changes of the spectral distribution, one considers that the energy of the process in certain frequency bands $[\lambda_j, \mu_j), 1 \le j \le J$, change suddenly. For any k and any $u \in [0,\pi]$ let

$$I_k(u) = \frac{1}{2\pi n_k} \left| \sum_{t=t_{k-1}+1}^{t_k} Y_t e^{itu} \right|^2 ,$$

be the peridogram of the sequence (Y_j) in the frequency band $[\lambda_j,\mu_j)$ and let

$$F_{kj} = \int_{\lambda_j}^{\mu_j} I_k(u)\, du$$

be the energy of $(Y_{t_{k-1}+1}, \ldots, Y_{t_k})$ in the frequency band $[\lambda_j,\mu_j)$. The contrast used for detecting the change-points is

$$C(y_{t_{k-1}+1}, \ldots, y_{t_k}) = -\frac{n_k}{n} \sum_{j=1}^{J} F_{kj}^2 .$$

The DCPC algorithm has been implemented in Matlab for different criteria: Changes in mean, variance, mean and variance, distribution function and spectra. It can be downloaded from M. Lavielle's personal webpage: http://www.math.u-psud.fr/~lavielle/programs/index.html.

THE SLEX METHOD

The auto-SLEX algorithm is a statistical procedure that automatically divides time series in segments that are approximately stationary and automatically chooses a smoothing parameter for the estimation of the spectrum that changes with time. The method is based on the SLEX (Smooth Localized complex EXponential) transform, which uses the SLEX vectors which are closely related to the classical Fourier transform. The method is presented in Ombao et al. (2002) and we follow here their presentation. The algorithms have been implemented in Matlab and are available in the web-page www.stat.uiuc.edu/~ombao.

As is well-known, Fourier functions are adequate for representing stationary random processes, since they are localized in frequency and the spectral properties of stationary processes are time-invariant, but they cannot represent processes with time-evolving spectral properties. To tackle the time localization problem, smooth compactly supported windows have been applied, but the functions resulting are no longer orthogonal. It is well-known that there does not exist a smooth window such that the windowed Fourier basis vectors are both orthogonal and localized in time and frequency. The SLEX functions avoid this problem using a projection operator, instead of a window, on the complex exponentials. It turns out that the action of the projection operator on a periodic function is equivalent to applying two especially constructed smooth windows to the Fourier basis functions.

The functions on the SLEX basis $\phi_\omega(u)$ are of the form

$$\phi_\omega(u) = \Psi_+(u)\exp(i2\pi\omega u) + \Psi_-(u)\exp(-i2\pi\omega u),$$

where $\omega \in [-1/2, 1/2]$ and $\Psi_+(u)$ and $\Psi_-(u)$ are specific smooth real valued functions that will be defined later. The SLEX basis functions have support on $[-\delta, 1+\delta]$, where $0 < \delta < 0.5$. Thus SLEX functions at different dyadic blocks overlap but they remain orthogonal.

The SLEX basis fun|ctions generalize directly to orthogonal SLEX basis vectors for representing time series. Let $a_0 < a_1$ be two integer time points, $|S| = a_1 - a_0$ and the overlap $\varepsilon = [\delta |S|]$, where $[\cdot]$ denotes the integer part. The support $\overline{S}$ of SLEX vectors on block S consists of time points defined on S and the overlap: $\overline{S} = \{a_0 - \varepsilon, \ldots, a_0, \ \ldots, a_0, \ldots, a_1 - 1, a_1 - 1 + \varepsilon\}$. A SLEX basis vector defined on block S has elements $\{\phi_{S,\omega_k,t}\}$ with

$$\begin{aligned}\phi_{S,\omega_k}(t) &= \phi_{\omega_k}((t-a_0)/|S|) \\ &= \Psi_{S,+}((t-a_0)/|S|)\exp\{i2\pi\omega_k(t-a_0)\} \\ &\quad + \Psi_{S,-}((t-a_0)/|S|)\exp\{-i2\pi\omega_k(t-a_0)\}\end{aligned}$$

where $\omega_k = k/|S|, k = -|S|/2+1, \ldots, |S|/2$. The windows can be represented in terms of a rising cut-off function r:

$$\Psi_{S,+}(t) = r^2\left(\frac{t-a_0}{\varepsilon}\right) r^2\left(\frac{a_1-t}{\varepsilon}\right),$$

$$\Psi_{S,-}(t) = r\left(\frac{t-a_0}{\varepsilon}\right) r\left(\frac{a_0-t}{\varepsilon}\right) - r\left(\frac{t-a_1}{\varepsilon}\right) r\left(\frac{a_1-t}{\varepsilon}\right)$$

In the specific implementation we use r is

$$r(u) = \sin\left(\frac{\pi}{4}(1+u)\right)$$

where $u \in [-1,1]$.

Auto-SLEX also uses the Best Basis Algorithm (BBA) of Coifman and Wickerhauser (1992) to choose the best segmentation using a cost function defined in terms of logarithms of the SLEX periodograms. First, the SLEX spectrum for the whole set is calculated, then the set is divided in two and the SLEX spectrum calculated for each half. The cost of each configuration is calculated and the algorithm chooses the lower cost configuration. This procedure goes on until one arrives at the best configuration or the minimum size for the intervals is reached.

The SLEX spectrum is calculated using the FFT algorithm and the set of data must have length a power of 2. Since the subintervals are obtained by successive divisions in two of the initial set, the length of all intervals obtained is also a power of 2, and their endpoints are sums of powers of 2. Since there is a minimum size for the intervals, related to the smallest set of data required to have a good estimation of the spectra, which in our case was set to $2^{10} = 1024$, there is a limit to the precision with which the algorithm can detect the change-points of a time series. This is a shortcoming of the SLEX algorithm. More details can be seen in Ombao et al. (2002).

ANALYSIS OF WAVE DATA FOR NORMAL SEA STATES

We consider two sets of data corresponding to three days in September 2005, starting at 0 h. on Sept. 1st, for two buoys deployed by the Costal Data Information Program, Integrative Oceanography Division, operated by the Scripps Institution of Oceanography (http://cdip.ucsd.edu/): Station 067 San Nicholas Island, off the coast of California, with a depth of 360 m. and Station 106 Waimea Bay, Hawai, with a depth of 200 m. In both cases data are sampled at a frequency of 1.28 Hz. We used both segmentation methods for these sets of data.

Station 067

We present in Figure 1 the data for station 067 along with the segments produced by both methods. In the upper half are the change-points obtained with the DCPC algorithm and in the lower half those produced by the SLEX method. The values are given in tables 1 and 2.

As can be seen from the graph and tables, the DCPC algorithm produced more segments: 45 vs. 27 and the change-points are placed at different instants. In fact, none of the 27 SLEX cuts have a DCPC cut within 5 minutes, 2 have one within 10 min., another 5 have one within 20 min. and 6 more have one within a half-hour. So in general the segmentations differ. Table 3 gives the basic statistical analysis of the length of the intervals with both algorithms.

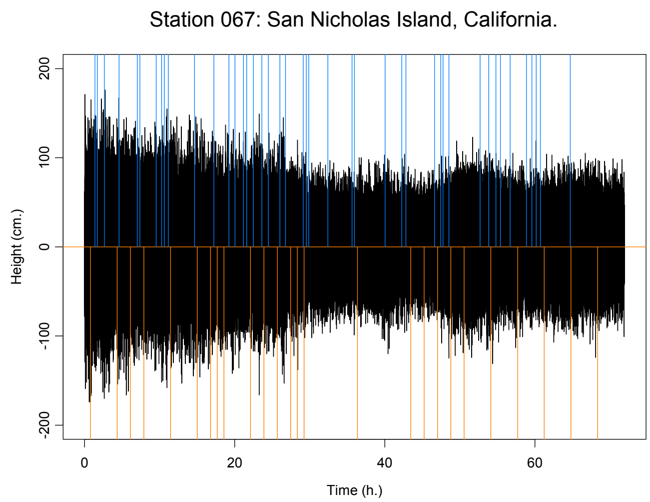

Figure 1. Wave height for Station 067. The DCPC segmentation is shown in the upper half, the SLEX segmentation is in the lower half.

Table 1. SLEX Change-points for Station 067 (min.).

00:46:57	4:20:17	6:06:57	7:53:37	11:26:57
15:00:17	16:45:31	17:40:17	18:33:37	22:06:57
23:53:37	25:40:17	27:26:57	28:20:17	29:13:37
36:20:17	43:26:57	45:13:37	47:00:17	48:46:57
50:33:37	54:06:57	57:40:17	61:13:37	64:46:57
68:20:17	71:53:37			

Table 2 DCPC Change-points for Station 067 (min.).

1:22:54	1:42:22	2:37:58	4:35:23	7:02:1
7:21:43	9:31:54	10:16:9	10:36:59	11:10:06
14:39:37	17:13:11	19:13:50	20:01:35	21:10:15
21:35:10	22:29:56	23:36:54	24:29:24	26:00:46
26:45:18	29:07:20	29:31:29	29:50:56	32:24:39
35:37:51	35:56:25	40:02:01	42:15:02	42:48:13
46:36:28	47:25:24	47:44:25	48:31:35	52:40:32
53:49:19	54:47:27	55:22:34	56:41:26	58:50:57
59:34:22	60:09:35	60:42:26	64:41:54	71:53:37

As can be seen from the graph and tables, the DCPC algorithm produced more segments: 45 vs. 27 and the change-points are placed at different instants. In fact, none of the 27 Slex cuts have a DCPC cut within 5 minutes, 2 have one within 10 min., another 5 have one within 20 min. and 6 more have one within a half-hour. So in general the segmentations differ. Table 3 gives the basic statistical analysis of the length of the intervals with both algorithms.

Table 3. Basic Statistics for Interval Length, Station 067 (min.).

	SLEX	DCPC
Min	49.95	9.42
1st. Qu	106.66	33.66
Median	106.66	56.86
Mean	159.76	93.77
3rd Qu.	213.33	132.31
Max	426.66	422.28
Var	9900.0	7224.1

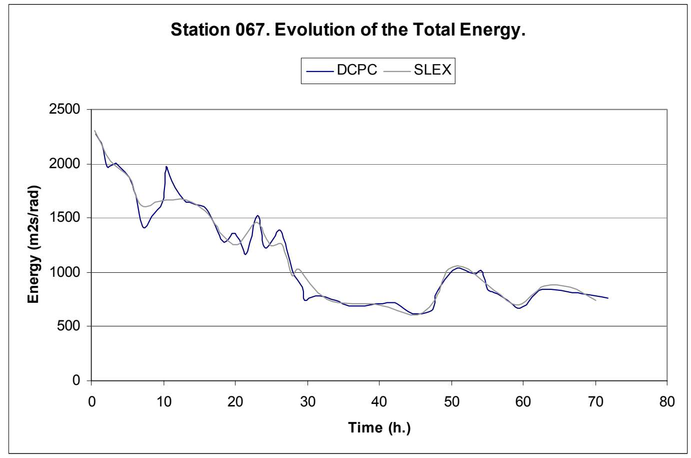

Figure 2. Evolution of the Total Energy for Station 067.

We calculated the spectra for each segment using the WAFO software, developed by Lund University of Technology, and studied the evolution of several properties of the spectra. The three properties we focused on were the total energy, the maximum value of the spectrum and the frequency corresponding to the maximum value (the dominant frequency). We present in figs. 2, 3 and 4 the evolution of these quantities as obtained for both segmentation methods. As can be seen from them, in general both curves follow similar patterns but since DCPC tends to produce smaller intervals, it detects changes that go unnoticed for the SLEX method. This can be seen in Figures 2 and 3 around 10 h. and in Figure 4 between 55 and 65 h.

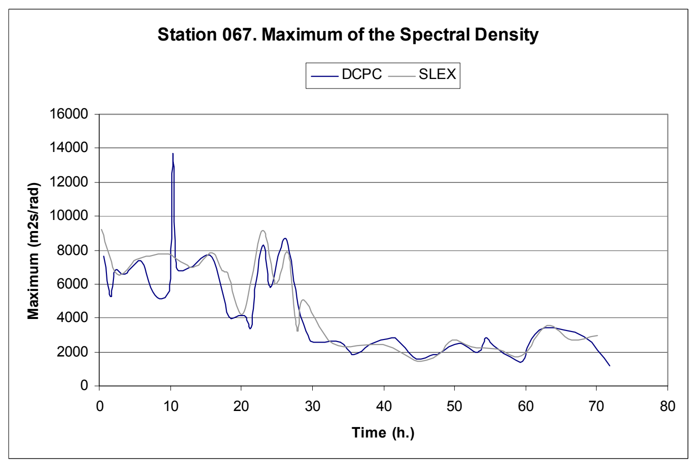

Figure 3. Evolution of the Maximum Value of the Spectral Density for Station 067.

Looking at Figure 1 and tables 1 and 2 one notices that intervals produced by the SLEX algorithm are frequently divided into smaller intervals by the DCPC method, but in some cases it is the other way round: e.g. the last DCPC interval is divided into 2 segments by SLEX. It is interesting to compare the spectra in these cases for both situations. We consider first the SLEX interval having endpoints 29:13:37.19 and 36:20:17.19 and the DCPC intervals having endpoints 29:07:20.63, 29:31:29.06, 29:50:56.25, 32:24:39.69, 35:37:51.88 and 35:56:25.94. The corresponding spectra are shown in Figure 5. Table 4 gives the values of the three properties considered before: total energy, maximum value and dominant frequency. The intervals are named SLEX1 and DCPCn with n=1,…5.

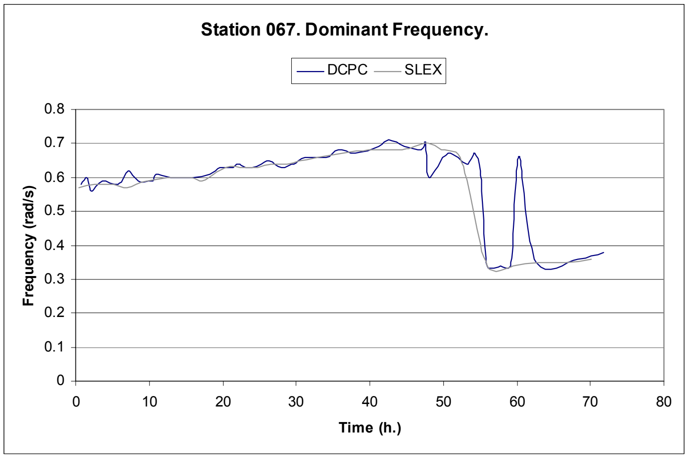

Figure 4. Evolution of the Dominant Frequency for Station 067.

As can be seen, the dominant frequency stays roughly constant throughout the interval, around 0.66. The total energy in DCPC1 is higher than the total energy in SLEX1 but then it decreases for the next 3 DCPC intervals, being roughly equal to the SLEX1 total energy, and in the last DCPC interval it decreases. The maximum value shows a similar pattern. Finally, there is a second peak in the SLEX1 spectrum that also appears in DCPC1, disappears in DCPC2, moves to a higher frequency in DCPC3, goes back to the same frequency in DCPC4 and disappears again in DCPC5.

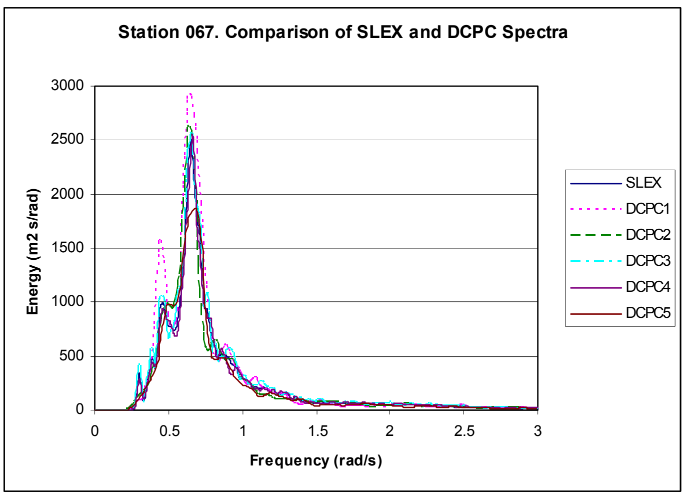

Figure 5. Comparison of SLEX and DCPC Spectra.

Table 4. Comparison of Spectral Properties.

Interval	Total Energy (m^2 s/rad.)	Max. Value (m^2 s/rad.)	Dom. Freq. 1 (rad/s)	Dom. Freq. 2 (rad/s)
SLEX1	752.8	2509.2	0.66	0.46
DCPC1	862.2	2936.6	0.64	045
DCPC2	738.6	2628.9	0.64	
DCPC3	784.3	2568	0.66	0.76
DCPC4	736.7	2549.4	0.66	0.46
DCPC5	688.4	1870	0.68	

It is interesting to note that DCPC5 is a short interval, lasting less than 19 min., which includes SLEX1's right endpoint.

Next we consider a DCPC interval divided in two by the SLEX algorithm. The DCPC interval has endpoints 64:41:54.84 and 71:53:37.19 while the SLEX endpoints are 64:46:57.19, 68:20:17.19 and 71:53:37.19. The corresponding spectra are shown in Figure 6. Table 5 gives the values of the total energy, maximum value and dominant frequency. The intervals are named SLEX1 and 2 and DCPC1.

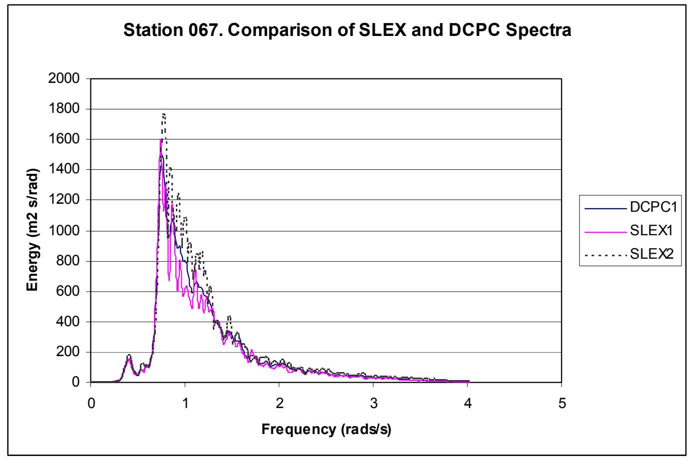

Figure 6. Comparison of SLEX and DCPC Spectra.

As can be seen from Figure 6 and Table 5, The SLEX spectra are rougher and for the first one the energy is higher than for the DCPC interval while for the second it is lower. The rest of the properties remain approximately constant.

Table 5. Comparison of Spectral Properties.

Interval	Total Energy (m^2 s/rad.)	Max. Value (m^2 s/rad.)	Dom. Freq. 1 (rad/s)	Dom. Freq. 2 (rad/s)
DCPC1	806.2	2892.4	0.36	0.71
SLEX1	865.0	2719.0	0.35	0.71
SLEX2	738.8	2965.8	0.36	0.70

Station 106

We did a similar analysis for the data from Station 106. Figure 7 shows the wave-height record along with the change-points determined by both algorithms. These change-points are listed in Tables 6 and 7.

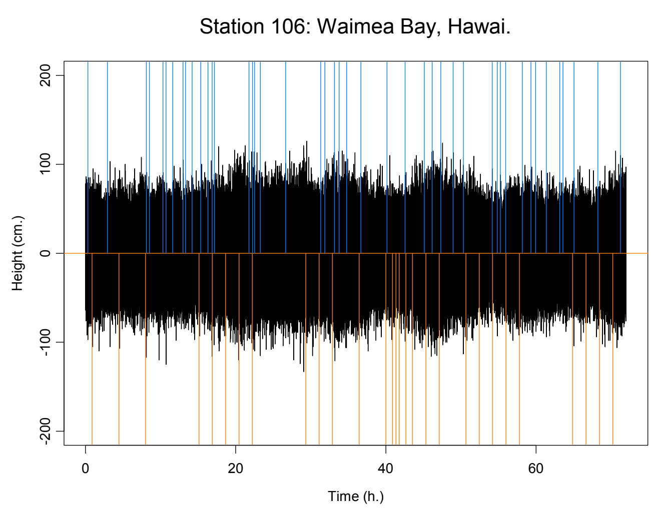

Figure 7. Wave height for Station 106. The DCPC segmentation is shown in the upper half, the SLEX segmentation is in the lower half.

Table 6. SLEX Cuts for Station 106 (min.).

0:53:20	4:26:40	8:00:01	15:06:40	16:53:20
18:40:00	20:26:40	22:13:20	29:20:00	31:06:40
32:53:20	36:26:40	40:00:00	40:53:20	41:20:00
41:46:40	42:40:00	43:33:20	45:20:00	47:06:40
50:40:00	52:26:40	54:13:20	56:00:00	57:46:40
64:53:20	66:40:00	68:26:40	70:13:20	

Table 7. DCPC Cuts for Station 106 (min.).

0:19:05	2:55:36	8:05:37	8:30:54	10:19:13
10:43:12	11:37:22	12:59:15	13:19:49	14:11:26
15:20:59	16:18:16	17:09:02	21:46:47	22:14:12
22:32:08	23:16:22	26:38:53	31:19:17	31:53:26
33:09:50	33:47:24	34:45:59	36:41:30	40:08:13
42:33:41	45:07:46	46:09:01	47:19:35	48:57:30
50:19:56	54:09:13	54:50:14	55:14:27	55:57:46
58:10:02	59:19:57	59:57:07	61:21:34	63:10:04
63:35:07	65:03:36	68:14:47	71:14:24	

Again, the DCPC algorithm proposes more changes than SLEX: 46 vs. 30. In this case 4 of the SLEX changes have a DCPC change point within 5 minutes, 6 have one within 10 min., 5 within 20 min. and 2 more within 30 min. So again the segmentations differ but not as markedly as before. Table 8 gives the basic statistics for interval length.

Table 8. Basic Statistics for Interval Length, Station 106

	SLEX	DCPC
Min	26.66	17.2
1st. Qu	106.66	37.26
Median	106.66	69.73
Mean	144.0	93.9
3rd Qu.	186.67	128.08
Max	426.67	310.01
Var	11946.6	5933.58

The next three figures give the evolution of the total energy, maximum of the spectral density and dominant frequency for Station 106. As can be seen, the remarks made in the previous case are valid again, although the differences are less marked.

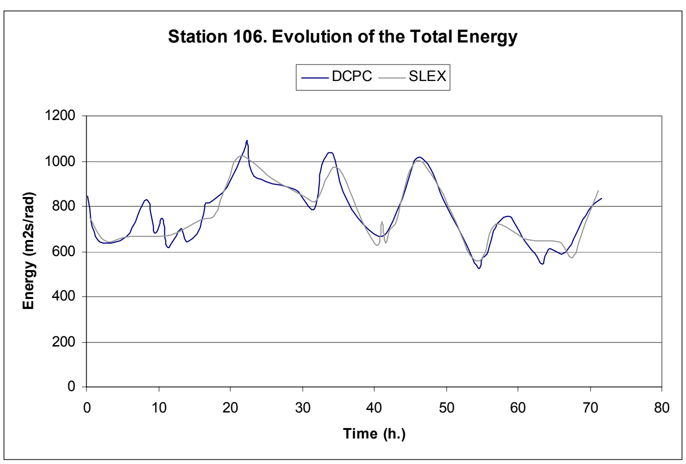

Figure 6. Evolution of the Total Energy for Station 106.

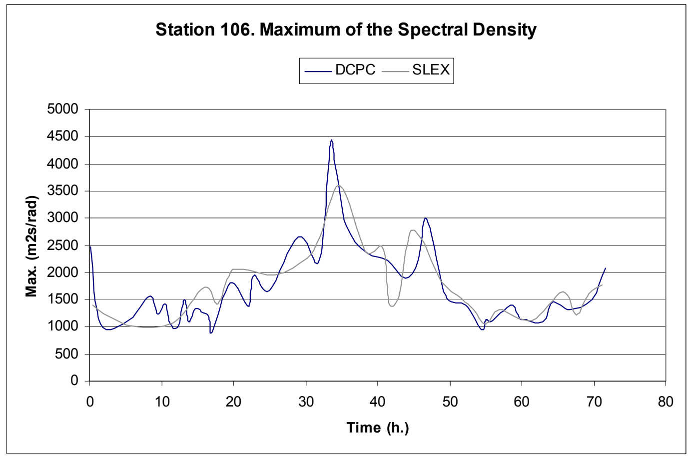

Figure 7. Evolution of the Maximum Value of the Spectral Density for Station 106.

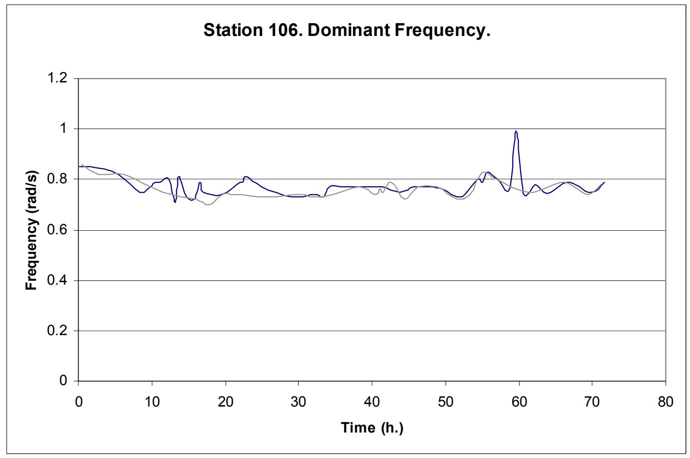

Figure 8. Evolution of the Dominant Frequency for Station 106.

HURRICANE CAMILLE DATA

As a final step in the comparison of the two methods we consider a highly non-stationary situation: we analyze the Hurricane Camille data. This set of data is well-known and has been previously considered by several authors (see, e.g. Forristall (1978) and Guedes Soares et al. (2004)).

Hurricane Camille occurred on August 17, 1969, was one of the strongest hurricanes to reach the USA coastline in the last century. It passed within 23 km. from a platform where a wave staff was measuring the wave height. This measuring device broke down around 4:30 pm on August 17 and the time series starts at 6 pm of the previous day, sampling at a rate of 1 Hz. We applied both methods to this data set and the results obtained are shown in Figure 9 and Tables 9-10.

Table 9. SLEX Change-points for Hurricane Camille (min.).

0:18:48	0:52:56	2:01:12	3:09:28	4:17:44
6:34:16	8:50:48	13:23:52	15:40:24	17:56:56
20:13:28				

Table 10. DCPC Change-points for Hurricane Camille (min.).

3:16:57	4:16:00	10:40:14	11:14:39	13:11:54
13:42:35	13:52:34	16:54:56	18:04:59	19:12:24
20:00:59	21:11:13	21:24:27		

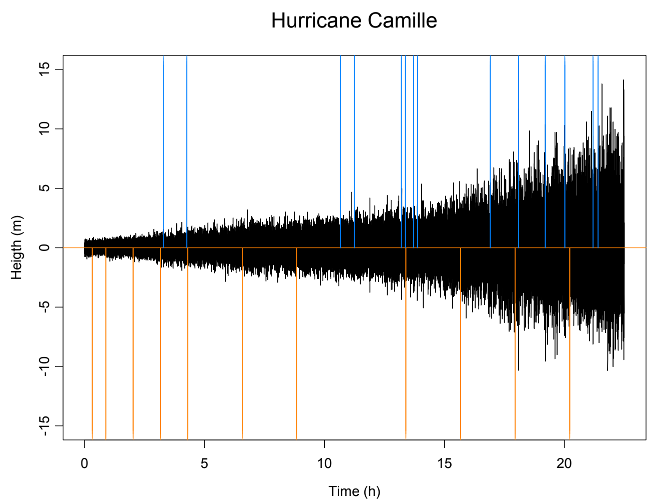

Figure 9. Wave height for Hurricane Camille. The DCPC segmentation is shown in the upper half, the SLEX segmentation is in the lower half.

This time the number of change-points is similar, 11 vs. 13, but the location is again different, except for 4 of the change-points which are reasonably close. The SLEX intervals tend to be evenly spaced while DCPC produces both very small and very large intervals. Table 11 gives the basic statistics for interval length. It can be seen from this table that the distribution for the SLEX intervals is more concentrated.

Table 11. Basic Statistics for Interval Length, Hurricane Camille (min.)

	SLEX	DCPC
Min	18.8	9.98
1st. Qu	68.27	27.36
Median	136.53	65.55
Mean	112.5	90.0
3rd Qu.	136.53	93.74
Max	273.07	384.23
Var.	4548.9	9888.4

We compare the evolution of the total energy, the maximum value of the spectral density and the dominant frequency for both methods. The results are shown in Figures 10-12.

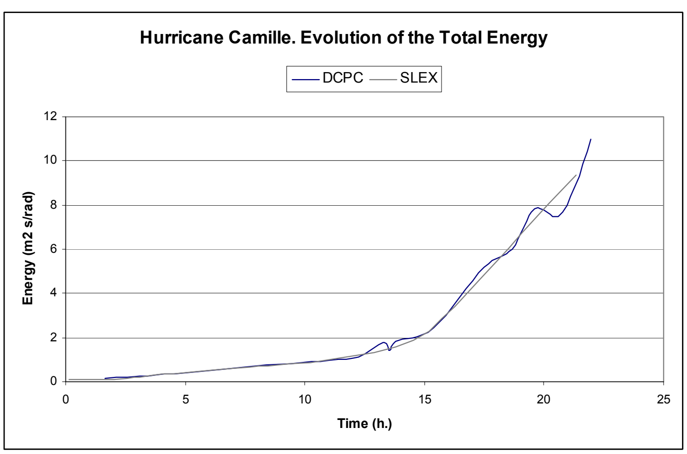

Figure 10. Evolution of the Total Energy for Hurricane Camille

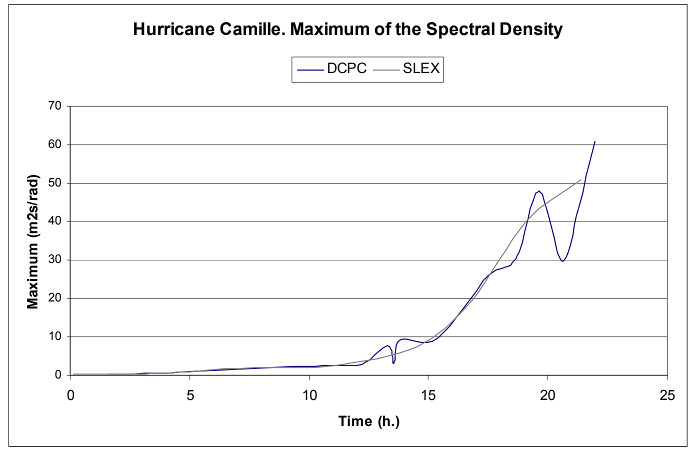

Figure 11. Evolution of the Maximum Value of the Spectral Density for Hurricane Camille.

Again, for each graph both curves show similar patterns, except at the end for Figures 10 and 11, where the SLEX algorithm fails to detect a change that occurs around 20 h. On the other hand, SLEX detects a change in the dominant frequency that occurs at the beginning while DCPC does not.

In Figure 9 one can see that the first DCPC interval in divided into 4 by the SLEX algorithm, while the last SLEX interval is divided into 3 subintervals. We now compare the corresponding spectra in both situations. We consider first the DCPC interval having endpoints 0 and 3:16:57, and the SLEX intervals having endpoints 0:18:48, 0:52:56, 2:01:12 and 3:09:28. The corresponding spectra are shown in Figure 13 and Table 12 gives the values for the three properties considered.

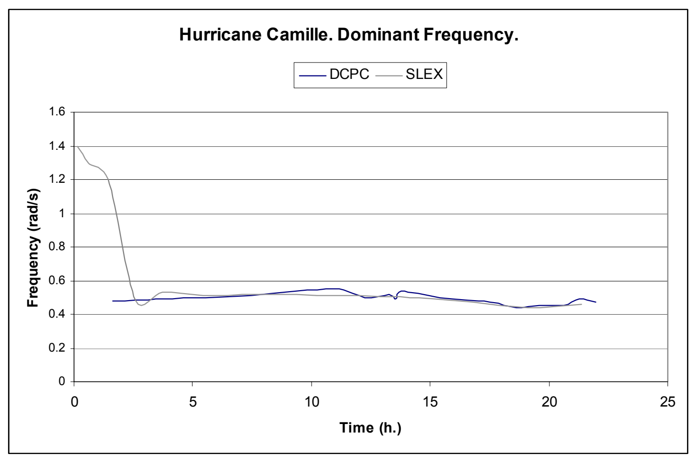

Figure 12. Evolution of the Dominant Frequency for Hurricane Camille.

Table 12. Comparison of Spectral Properties.

Interval	Total Energy (m^2 s/rad.)	Max. Value (m^2 s/rad.)	Dom. Freq. 1 (rad/s)	Dom. Freq. 2 (rad/s)
DCPC1	0.127	0.159	0.48	1.2
SLEX1	0.0779	0.157	1.4	1.2
SLEX2	0.0791	0.165	1.3	1.2
SLEX3	0.111	0.168	1.2	0.46
SLEX4	0.169	0.382	.48	1.2

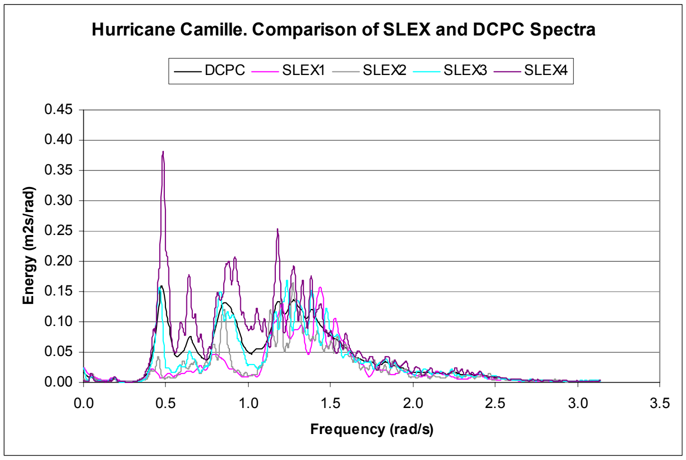

Figure 13. Comparison of DCPC and SLEX Spectra for the First DCPC interval.

As can be seen, in the SLEX spectra the dominant frequency moves down and for the last interval it coincides with the dominant frequency for the DCPC interval. All five spectra have approximately the same two dominant frequencies, 0.48 and 1.2 rads/s, but their relative importance is different. The graph shows that all the spectra have at least 4 peaks.

The total energy and the maximum vale also change, the SLEX spectra energy grows and the DCPC value is in-between the third and fourth value for the SLEX spectra. The maximum value for the fourth SLEX spectra is twice the size of the rest.

The last SLEX interval, with endpoints 20:13:28 and 22:30:00, is divided into 3 subintervals by the DCPC algorithm, having endpoints 20:00:59, 21:11:13, 21:24:27 and 22:30:00. The spectra are shown in Figure 14 and Table 13 gives the values of the spectral properties.

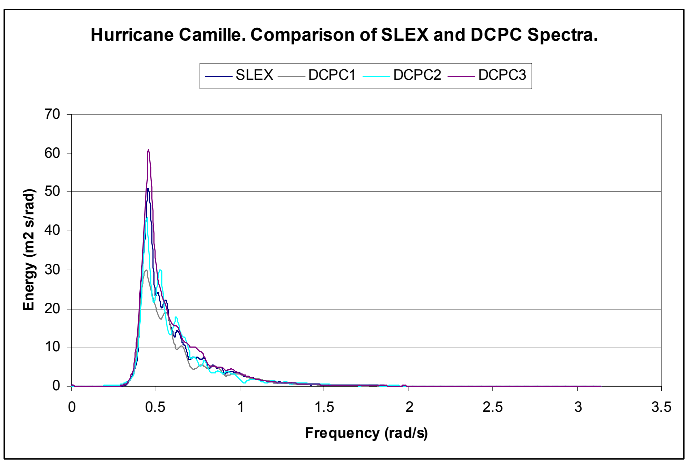

Figure 14. Comparison of SLEX and DCPC Spectra.

Table 13. Comparison of Spectral Properties.

Interval	Total Energy (m^2 s/rad.)	Max. Value (m^2 s/rad.)	Dom. Freq. 1 (rad/s)	Dom. Freq. 2 (rad/s)
SLEX1	6.35	50.94	0.46	
DCPC1	7.47	29.83	0.45	0.43
DCPC2	8.84	43.44	0.49	
DCPC3	10.982	60.93	0.47	

In this case the dominant frequency stays approximately constant and the main change occur in the energy, reflected both by the change in total energy and the maximum value of the spectral density. The shape of the different spectra is similar.

CONCLUSIONS

We have considered two methods for detecting change-points in a time series: Detection of Changes by Penalized Contrasts (DCPC) and Smooth Localized complex EXponentials (SLEX). These algorithms were tried on three set of data two of them coming from 'normal' sea states (stations 067 and 106) and one from a hurricane.

The results obtained by these methods differ. In normal conditions DCPC tends to produce more change-points and hence smaller stationarity intervals. For hurricane data the number of change-points is approximately the same, although SLEX produced intervals that were more uniform in length.

For each interval we estimated the spectral density and also analyzed the evolution of some spectral characteristics: the total energy, the maximum of the spectral density and the dominant frequency. The general pattern of evolution is the same for both methods in all three cases but DCPC seem to capture more variation than SLEX. This is probably due to the fact that it produces more change-points.

In some cases we also compared the spectra for intervals that were subdivided by the other method.

SLEX is fast and easy to use and gives more information than we have used here (SLEX spectra are automatically calculated and stored, and a graph of frequency vs. time is given where different 'intensities' for the frequencies are color-coded. This graphs gives an idea of the spectral evolution of the data being analyzed). DCPC is slower and cannot handle very large sets of data, but on the other hand it is not restricted to finding intervals having lengths a power of 2. This accounts for the longer time it takes to analyze the data sets.

In our view none of the methods seemed completely satisfactory but this is probably not a flaw of the segmentation methods considered but of the basic assumption that changes in the wave pattern occur abruptly.

It is important to remark that our conclusions are based on only 3 data sets. Further research is required to validate our conclusions, specially regarding hurricane conditions.

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