Flexible multi-robot formation tracking: a practical evaluation on DDR's

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Abstract: This paper presents a flexible multi-robot formation tracking control that effectively combines a trajectory tracking component with a consensus-based one. Stability is guaranteed despite the number of agents, the dimension of the agent's state and the connectivity between them. The proposed approach was implemented an evaluated on homemade differential-drive robots (DDR's). The controller's performance is compared under three connectivity topology: fully connected, ring and spanning tree; the fully connected topology gives the best performance and the spanning tree the worst. Additionally, the consensus and non-consensus controllers are compared, indicating that the proposed approach achieves better performance despite perturbations.

Keywords: Mobile Robots, Networked robotic systems, Tracking, Vehicle dynamic systems, Distributed Control, Multi-agent systems

1. INTRODUCTION

A multi-robot system (MRS) outperforms the capabilities of a single robot by using coordination and collaboration. Some applications of MRS's are environmental sensing (Fan et al., 2018), coverage control for precision agriculture (Davoodi et al., 2018) and object transportation (Hu et al., 2021), to give some examples. Decentralized or distributed control methods are preferred due to their robustness and requirements of local measurements between robots, where consensus theory is the backbone of such kind of methods (Olfati-Saber et al., 2007). Consensus control has been extended to achieve formations by imposing some constant deviations for the relative robot's positions (Oh et al., 2015). In some applications it is required that the whole formation moves tracking a predefined trajectory specified by a leader robot (Ren and Sorensen, 2008).

A challenging aspect of formation control is to consider time-varying formation tracking (TVFT), which means that the formation can change according to time-parametrized functions that specifies the desired relative robot's positions. This problem has been addressed in (Zhou et al., 2020) for heterogeneous swarm systems and relying on a distributed observer to estimate the leader's state. In (Dong and Hu, 2017), TVFT for linear multiagent systems with multiple leaders is studied. For wheeled robots, a distributed model predictive control strategy based on consensus is proposed in (Xiao and Philip Chen, 2021). The TVFT problem is addressed using a consensus-based approach in (Santiaguillo-Salinas and Aranda-Bricaire, 2017); the whole formation can be scaled as defined by a time-varying function and also collision avoidance is considered. To the authors' knowledge, existing methods of TVFT are limited to deal with simple trajectories and only scaling of the formation can be achieved.

This article presents a flexible multi-agent formation control that considers TVFT, which achieves stability and good performance. It is flexible since it is capable of tracking formations that change size, orientation and position along time. The control structure is simple and its convergence is reached independently from the number of agents, the state dimension and the connection topology. To compare the controller performance experimental results are provided using differential-drive robots and testing different connectivity topologies.

The organization of the paper is as follows: Section 2 defines the multi-agent formation tracking problem and presents the proposed controller. Section 3 conducts the error and stability analysis. The implementation of the controller and experimental results are show in section 4 and the paper closes with some conclusions in section 5.

2. PROBLEM STATEMENT AND CONTROL LAW

A first order dynamic multi-agent system (MAS) is considered in this work as follows:

$$\dot{\xi}_i = u_i, \tag{1}$$

$\xi_i = [x_i, y_i]^T \in \mathbf{R}^2$ is the position vector, $i = 1, 2, \ldots, n$ represent the number of agents, and $u_i = [v_x, v_y]^T$ is the control input, and corresponds to the velocity vector for

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each agent. The aim is that each agent's position ξ_i and velocity $\dot{\xi}_i$ track asymptotically desired values $(\xi_{iD}, \dot{\xi}_{iD})$. To this end, it is proposed the following controller:

$$u_i = u_{ip} + u_{ic}, \quad (2)$$
$$u_{ip} = \dot{\xi}_{iD} - \alpha(\xi_i - \xi_{iD}), \quad (3)$$
$$u_{ic} = -\sum_{j=1}^{n} a_{ij}[(\xi_i - \xi_{iD}) - (\xi_j - \xi_{jD})], \quad (4)$$

where $\alpha > 0$ is the proportional gain for the position error, u_{ip} is the trajectory tracking controller, and u_{ic} is the consensus controller. When agents i and j are connected $a_{ij} = 1$, which is encoded in the adjacency matrix $\mathcal{A}$, moreover $\xi_{iD} \neq \xi_{jD}$. The stability analysis of controller (2) is depicted in the next section.

3. ERROR DYNAMICS AND STABILITY

The i_{th} agent's error and its derivative is as follows:

$$e_i = \xi_i - \xi_{iD}, \quad (5)$$
$$\dot{e}_i = \dot{\xi}_i - \dot{\xi}_{iD}. \quad (6)$$

By replacing (5) and (6) in (2) it is obtained,

$$\dot{\xi}_i = \dot{\xi}_{iD} - \alpha e_i - \sum_{j=1}^{n} a_{ij}(e_i - e_j), \quad (7)$$

after rearanging (7) and considering (6), the closed loop error dynamics is:

$$\dot{e}_i = -\alpha e_i - \sum_{j=1}^{n} a_{ij}(e_i - e_j). \quad (8)$$

Aditionally, for n agents with dimension d it is obtained the following matrix expression:

$$\dot{e} = ((-\alpha I_n - \mathcal{L}) \otimes I_d)e, \quad (9)$$

I_n and I_d are identity matrices with n and d dimension respectively, for instance, when $d = 3$, $e_i = [e_{ix}, e_{iy}, e_{iz}]^T$, and $\mathcal{L}$ is the *Laplacian* matrix that depends on the adjacency matrix $\mathcal{A}$ with components a_{ij}.

3.1 Stability analysis

Proposition 1. If the matrix $((-\alpha I_n - \mathcal{L}) \otimes I_d)$ in (9) is Hurwitz guarantees asymptotic convergence of the error vector (e) to zero.

Definition 1 (G.W. Stewart, 1990). Given a $n \times n$ matrix B, with elements b_{ij}, every eigenvalue of B lies within at least one of the Gershgorin's discs $D(b_{ii}, R_i)$, which are defined as a circle located in the complex plane with center b_{ii} along the real axis and a radius $R_i = \sum_{j=1, j\neq i}^{n} |b_{ij}|$.

Consider $-\mathcal{L}$ in (9), the associated *Gershgorin's* disks $D(b_{ii}, R_i)$ center is defined by $b_{ii} = -\sum_{j=1, j\neq i}^{n} a_{ij}$, for $i = 1 \dots n$. The radius of each Gershgorin disk is $R_i = \sum_{j=1, j\neq i}^{n} |a_{ij}|$, and the disks $D(b_{ii}, R_i)$, depicted in figure 1, are tangent to the imaginary axis and located on the left side of the complex plane, showing that the matrix $-\mathcal{L}$ is semidefinite negative under any connectivity.

The matrix $(-\alpha I_n - \mathcal{L})$ Gershgorin's disks are displaced to the left of the imaginary axis by α units, therefore we can conclude that this matrix is Hurwitz. As a result, the

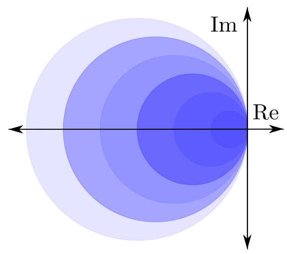

Fig. 1. $-\mathcal{L}$ matrix Gershgorin disks for $n = 6$.

controller (2) in closed loop with system (1) for $\xi_i \in \mathbf{R}^d$, ensures asymptotic error convergence to zero.

4. EXPERIMENTAL RESULTS

To test the controller (2), differential-drive mobile robots (DDR's) are proposed as agents for the MRS.

4.1 Implementation

The kinematic model for the agents is:

$$\begin{bmatrix} \dot{\xi}_{xi} \\ \dot{\xi}_{yi} \\ \dot{\theta}_i \end{bmatrix} = \begin{bmatrix} \cos(\theta_i) & 0 \\ \sin(\theta_i) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix}, \quad (10)$$

where ξ_{xi}, ξ_{yi} and θ_i are the states of the i^{th} robot with respect to its center of mass, v_i is the displacement velocity and ω_i is the angular velocity of the i^{th} robot. To control each agent, a control point P_i is selected (see figure 2); the kinematics of this point is:

$$\begin{bmatrix} \dot{x}_{Pi} \\ \dot{y}_{Pi} \end{bmatrix} = \begin{bmatrix} \cos\theta_i & -\ell\sin\theta_i \\ \sin\theta_i & \ell\cos\theta_i \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix}. \quad (11)$$

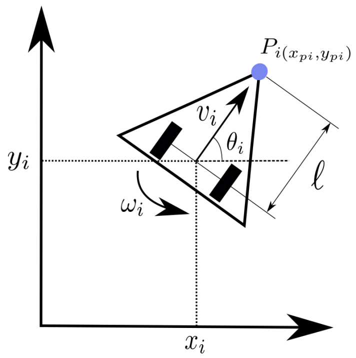

Fig. 2. DDR model and the control point P_i.

The angular velocities of the left (ω_l) and right (ω_r) wheels are:

$$\omega_l = \frac{2v_i - \omega_i L}{2R}, \qquad \omega_r = \frac{2v_i + \omega_i L}{2R}, \quad (12)$$

L is the distance between wheels and R is the wheels' radius.

The robots used for experimentation are named *Mitotiani v1* (from the Nahuatl *dancer*, designed and built in Cinvestav Saltillo, see figure 3). These robots receive angular velocity commands for every wheel via Bluetooth, an on-board micro-controller executes a *PID* control for each wheel. The wheels angular speed feedback signals are obtained from 12 pulses per revolution (ppr) encoders that in combination with a 78.125:1 gearbox achieve a 937 ppr resolution.

Fig. 3. Differential-drive robot made in Cinvestav.

Data acquisition. To measure every agent's position and orientation *ArUco* markers where used in combination with a 30 frames per second and 1280x720 pixel USB camera, the work space is a 4.13 by 2.32 meters plane; see figure 4.

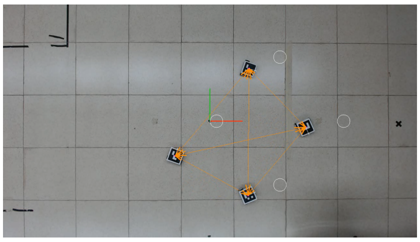

Fig. 4. Workspace provided by the camera field of view.

4.2 Experiments

A complex time-varying formation in the plane is assigned (see figure 5), with a virtual center $VC(x_c, y_c)$. The ξ_{iD} components are defined as follows:

$$\begin{aligned} \xi_{iDx} &= x_c + r\cos(\theta + \phi_i), \\ \xi_{iDy} &= y_c + r\sin(\theta + \phi_i). \end{aligned} \tag{13}$$

Where θ indicates the formation orientation and r is the distance between VC and the agents. The angle ϕ_i defines the desired position of the i^{th} agent along a circumference.

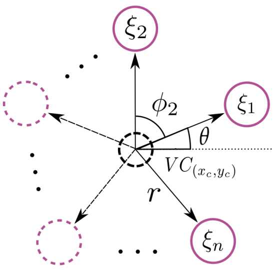

Fig. 5. Formation definition for n agents.

Given (13), the desired velocities are:

$$\begin{aligned} \dot{\xi}_{iDx} &= \dot{x}_c + \dot{r}\cos(\theta + \phi_i) - r\dot{\theta}\sin(\theta + \phi_i), \\ \dot{\xi}_{iDy} &= \dot{y}_c + \dot{r}\sin(\theta + \phi_i) + r\dot{\theta}\cos(\theta + \phi_i). \end{aligned} \tag{14}$$

In this experiment VC is defined as:

$$\begin{aligned} x_c &= 0.7\cos(0.2t), \\ y_c &= 0.7\sin(0.2t). \end{aligned} \tag{15}$$

The formation radius for the experiment is 0.3 meters and its angular velocity $\dot{\theta}$ is 0.6 radians/s.

4.3 Results

Each experiment has two stages, in the first one, the agents move from random positions to the formation (5s), the second stage executes the formation path tracking.
Four agents and three different topologies, were tested: fully connected, ring and spanning tree, with $\alpha = 0.25$, video experiments can be accessed in *link*. The desired and measured trajectories for a fully connected agent network are displayed in figure 6; the four agents follow their desired trajectories.

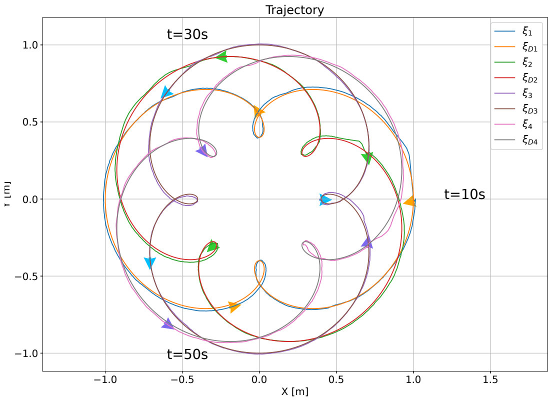

Fig. 6. Desired and measured trajectories for a fully connected network.

The error norm for fully connected, ring and tree topologies are displayed in figures 7, 8 and 9, respectively. The three figures show asymptotic convergence in the first stage of the experiment ($t < 5s$).

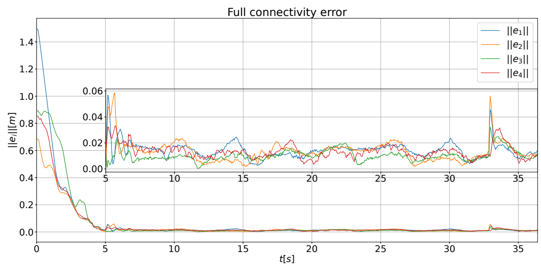

Fig. 7. Error details for the fully connected topology.

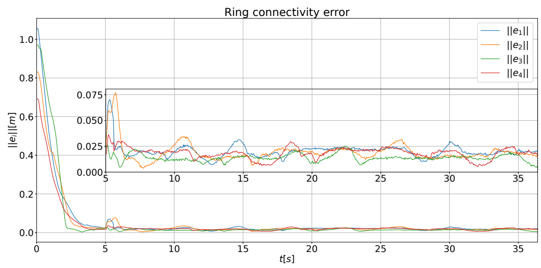

Fig. 8. Error details for the ring topology.

The mean error norm for the three topologies and every agent are depicted in table 1. It can be noticed that the fully connected topology achieved best performance,

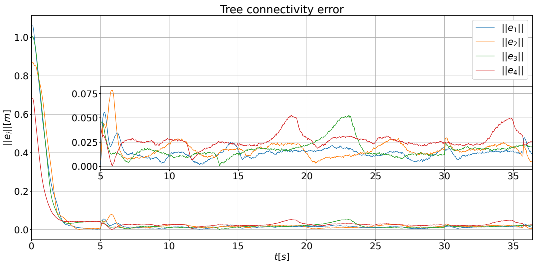

Fig. 9. Error details for the tree topology.

Table 1. Mean Euclidean error norm for every agent and topology.

agent/connectivity	Full	Ring	Tree
Agent 1	0.0143	0.0202	0.0141
Agent 2	0.0135	0.0192	0.0180
Agent 3	0.0102	0.0138	0.0178
Agent 4	0.0133	0.0185	0.0266
Formation	0.0128	0.0179	0.0191

followed by a ring connectivity, meanwhile the expanded tree topology achieves the poorest performance.

An experiment was performed with $u_{ic} = 0$. The errors are shown in figure 10, the mean error norm for the formation is 0.0443 meters, which is bigger than the obtained by the tree connectivity. Finally, in another experiment a perturbation was induced by changing $\xi_{3D} = [0, 0]^T$ for $18 < t < 19s$ for both, consensus and non-consensus approach, see figure 11. A faster recovery is obtained in the experiment using consensus; the error norm along the experiment is 0.022 and 0.030 meters, for the consensus and non-consensus approach, respectively.

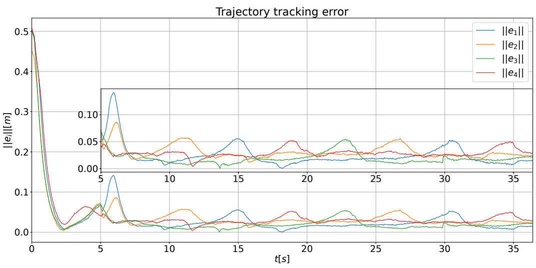

Fig. 10. Error details for the path tracking controller.

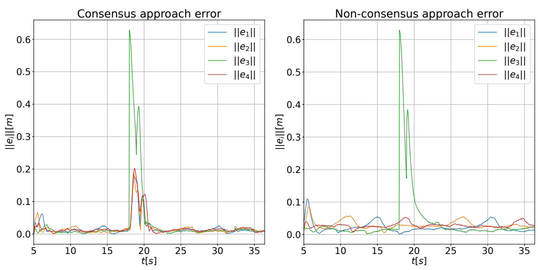

Fig. 11. Error norm comparison between consensus and non consensus approach controllers.

5. CONCLUSIONS

We proposed a simple yet effective multi-agent control law, composed by two parts: *trajectory tracking* and *consensus*. The proposed control law has high flexibility since can achieve formation tracking for any number of agents in arbitrary dimension.
The provided stability analysis guarantees asymptotic convergence for $\alpha > 0$ regarding the connectivity topology. Due to the *Gershgorin's circle theorem*, it can be ensured that the eigenvalues of controller's error dynamics matrix are located on the left-hand side of the complex plane, and are displaced α units from the imaginary axis.
The implementation of the controller on DDR's shows a simple method to map the control input u_i in Cartesian coordinates to the robot's wheels angular velocity. A controller performance comparative under different conectivities was provided, showing formations with higher algebraic connectivity (i.e. fully connected) as the best and low algebraic connectivity (i.e. spanning tree) as the worst.

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