

Weakly-Calibrated Visual Control of Mobile Robots using the Trifocal Tensor and Central Cameras

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Abstract—In this paper, we present the synthesis of two control schemes that exploit the properties of the trifocal tensor computed from bearing measurements (1D TT), for the pose-regulation problem of mobile robots. Both control schemes are valid for vision systems obeying a central projection model, in such a way that visibility constraint problems can be overcome. The use of the 1D TT avoids the need of a complete camera calibration for any type of central camera, so that, weakly-calibrated control schemes are obtained. This benefit of the 1D TT as measurement is exploited in an image-based (IB) approach as well as in a position-based (PB) approach. The IB scheme employs direct feedback of the elements of the tensor without commuting to any other approach during the control task. The PB approach relies on the feedback of the pose estimated dynamically from the 1D TT. Both visual control schemes are evaluated through real-world experiments using a hypercatadioptric imaging system.

I. INTRODUCTION

Visual control of mobile robots is an interesting research field, motivated by the introduction of this type of robots as *service robots*. Particularly, wheeled mobile robots (WMR) are well appreciated in service tasks, where the positioning at a desired location is an important aspect. This paper describes an approach to drive a WMR equipped with a central generic camera onboard to a desired location, which is specified by a target image previously acquired, i.e., using a teach-by-showing strategy. Along the years, the research on visual control has dedicated important efforts to find suitable error functions in order to obtain a desired behavior of the robotic system in terms of stability and robustness of the closed loop control. The basic approaches are typically separated in image-based (IB) schemes, in which the error function consists of a set of features that are directly available in the image data, and position-based (PB), in which a set of 3D parameters must be estimated from image measurements [1].

The goal of steering a robot to a desired location by visual servoing (VS) is carried out by minimizing an error function that relates visual data, typically from two images: the current and the target one. We propose to take advantage of more information by using three views and the geometric constraint that describes the complete geometry between them, the trifocal tensor (TT). This geometric constraint is more robust and more stable than those based on two views as it is also independent of the observed scene [2]. Its simplified version constrained to planar motion, the 1D TT,

has proved its effectiveness for localization in [3] and [4], but has been less studied for control applications. In these works, conventional perspective and omnidirectional cameras are converted to 1D virtual cameras through a transformation of bearing measurements. The authors of [5] assert that the radial 1D camera model is sufficiently general to represent the great majority of omnidirectional cameras under the assumption of knowing the center of radial distortion. The 2D TT has been introduced for visual control of mobile robots in [6]. This approach shows good results reaching the target location, but it uses a non-exact system inversion that suffers of potential stability problems. An application of the TT related to camera-motion estimation is presented in [7]. It introduces a filtering algorithm with the TT as measurement model to tackle the vision-based pose-tracking problem for augmented reality applications. The use of more than two views in VS provides robustness and enough information to correct also depth from visual feedback, which is not possible from two views.

In this paper, we present two visual control schemes that exploit the property of the 1D TT of being estimated from bearing information. This provides the advantage that parameters related to focal length do not appear in the control laws, so that, weakly-calibrated schemes are obtained. Additionally, the simplified representation of the imaging systems as 1D virtual cameras provides the versatility of the schemes to be applied using any central camera [8]. First, an IB scheme that uses direct feedback of the elements of the 1D TT is presented, as a summary of our previous work [9]. The proposed switching control law turns out to be a square control system that consists of two controllers, which correct position and orientation in two steps. Secondly, we present a PB scheme that feeds back the robot pose estimated dynamically from the 1D TT, which has been introduced in [10]. We show the property of observability of the system with the 1D TT as measurement using linear theory. The proposed PB scheme corrects the robot position and orientation using smooth robot velocities from a single control law. Real-world experiments using a hypercatadioptric imaging system as sensor show the validity of the proposed approaches.

The paper is organized as follows. Section II specifies the mathematical modeling of the mobile robot and the 1D TT geometric constraint. Section III presents the development of the image-based approach and Section IV describes the position-based approach. Section V shows the performance of the proposed approaches via real-world experiments. Finally, Section VI provides the conclusions.

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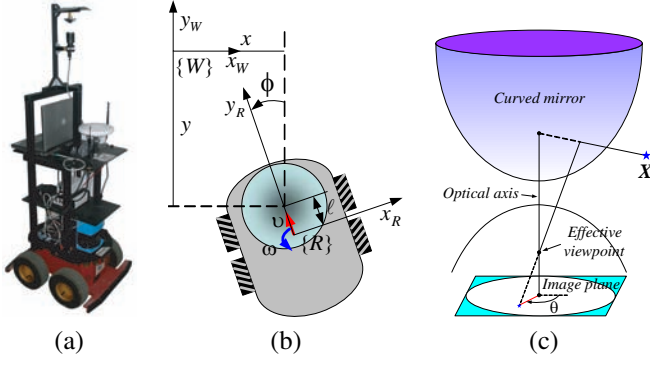


Fig. 1. Description of the robot-camera configuration. (a) Our robotic platform with an hypercatadioptric camera on top. (b) Robot frame definition. (c) A generic central catadioptric system.

II. MATHEMATICAL MODELING

A. Robot Model

This work focuses on controlling a wheeled mobile robot through the information given by a generic central camera mounted onboard, as shown in Fig. 1(a), and under the framework that is depicted in Fig. 1(b). The camera can be eventually translated a distance ℓ along the longitudinal axis of the robot. The kinematic motion model of the camera-robot system as expressed in state space is

$$\begin{aligned}\dot{x} &= -\omega \ell \cos \phi - v \sin \phi, \\ \dot{y} &= -\omega \ell \sin \phi + v \cos \phi, \\ \dot{\phi} &= \omega.\end{aligned}\quad (1)$$

By applying an Euler approximation (forward difference) on the continuous derivatives, the discrete version of the camera-robot model is obtained:

$$\begin{aligned}x_{k+1} &= x_k - T_s (\omega_k \ell \cos \phi_k + v_k \sin \phi_k), \\ y_{k+1} &= y_k - T_s (\omega_k \ell \sin \phi_k - v_k \cos \phi_k), \\ \phi_{k+1} &= \phi_k + T_s \omega_k,\end{aligned}\quad (2)$$

where T_s is the sampling period. In the sequel, we use the notation $s\phi = \sin \phi$, $c\phi = \cos \phi$.

B. The Trifocal Tensor for Central Cameras

The procedure to estimate the trifocal tensor (TT) is basically the same for conventional and central catadioptric cameras if it is formulated in terms of rays that emanate from the effective viewpoint [8]. In the case of planar motion, the simplified version of the tensor, the 1D TT, particularly adapts to the property of omnidirectional images to preserve bearing information regardless of the high radial distortion induced by lenses and mirrors. Fig. 1(c) shows the bearing angle of an observed feature in a hypercatadioptric system looking upwards. The angle is measured with respect to a frame centered in the principal point of the image. Therefore, the bearing measurement θ can be converted to its 1D projection as $\mathbf{p} = (\sin \theta, \cos \theta)^T$. For conventional cameras looking forward, the projective formulation can be obtained using the normalized x -coordinate of the point features with respect to the principal point, i.e., $\mathbf{p} = (u_n, 1)^T$. By relating

this representation for three different views of a feature, it results in the trifocal constraint

$$\sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 T_{ijk} u_i v_j w_k = 0, \quad (3)$$

where $\mathbf{u} = (u_1, u_2)^T$, $\mathbf{v} = (v_1, v_2)^T$ and $\mathbf{w} = (w_1, w_2)^T$ are the image coordinates of a feature projected in the 1D virtual retina of the first, second and third camera respectively, and T_{ijk} are the eight elements of the 1D TT.

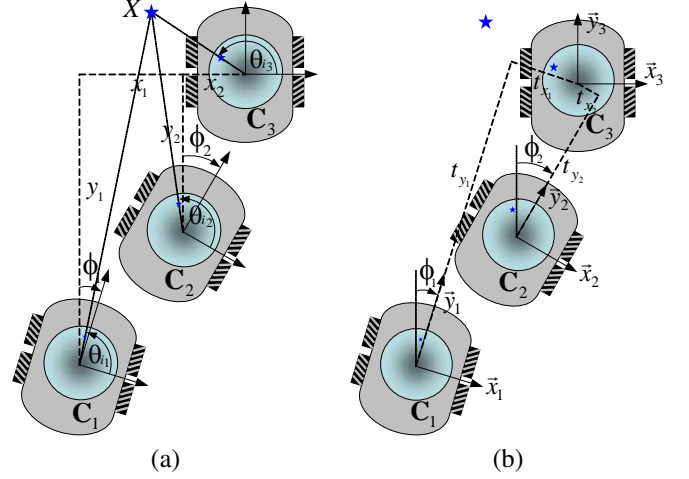


Fig. 2. Framework of the three-view geometry. (a) Global reference definition with origin in the third view and bearing measurements θ of a feature. (b) Relative location between cameras with a fixed reference frame on each view.

Let us define a global reference frame as depicted in Fig. 2(a) with the origin in the third camera. Then, the camera locations with respect to that global reference are $\mathbf{C}_1 = (x_1, y_1, \phi_1)$, $\mathbf{C}_2 = (x_2, y_2, \phi_2)$ and $\mathbf{C}_3 = (x_3, y_3, \phi_3) = (0, 0, 0)$. The relative locations between cameras are defined by a local reference frame in each camera as shown in Fig. 2(b). The geometry of the three views is encoded in the tensor elements as follows:

$$\mathbf{T}_{ijk}^m = \begin{bmatrix} T_{111}^m \\ T_{112}^m \\ T_{121}^m \\ T_{122}^m \\ T_{211}^m \\ T_{212}^m \\ T_{221}^m \\ T_{222}^m \end{bmatrix} = \begin{bmatrix} t_{y_1} s\phi_2 - t_{y_2} s\phi_1 \\ -t_{y_1} c\phi_2 + t_{y_2} c\phi_1 \\ t_{y_1} c\phi_2 + t_{x_2} s\phi_1 \\ t_{y_1} s\phi_2 - t_{x_2} c\phi_1 \\ -t_{x_1} s\phi_2 - t_{y_2} c\phi_1 \\ t_{x_1} c\phi_2 - t_{y_2} s\phi_1 \\ -t_{x_1} c\phi_2 + t_{x_2} c\phi_1 \\ -t_{x_1} s\phi_2 + t_{x_2} s\phi_1 \end{bmatrix} \quad (4)$$

where $t_{x_i} = -x_i c\phi_i - y_i s\phi_i$, $t_{y_i} = x_i s\phi_i - y_i c\phi_i$ for $i = 1, 2$. Some details on deducing the trifocal constraint (3) and the expressions in (4) can be found in [4]. Additional constraints $-T_{111} + T_{122} + T_{212} + T_{221} = 0$, and $T_{112} + T_{121} + T_{211} + T_{222} = 0$ are accomplished when the 1D TT is computed from a calibrated retina. These calibration constraints allow us to estimate the 1D TT from only five triplets of point correspondences, which improves the estimation [4]. It is worth noting that these additional constraints can be always used for central cameras, because

the bearing measurements are independent on focal length. Only the center of projection for omnidirectional images or the principal point for conventional cameras is required to estimate the 1D TT. Thus, the use of this tensor as measurement results in weakly-calibrated control schemes, in contrast to previous approaches [11], [12], [13].

It is worth mentioning that it is needed to normalize the tensor elements in order to fix a scale of the measurements, where normalize means to divide each element by one of them that can be assumed as constant (T_{121}).

III. IMAGE-BASED CONTROL FROM THE 1D TT

The problem of taking three variables to desired values $(x_2, y_2, \phi_2) = (0, 0, 0)$ may be solved with at least three outputs being controlled, but defining more than two outputs generate a non-square dynamic system, in which its non-invertibility makes difficult to prove stability. The trifocal tensor is an overconstrained measurement; however, it is possible to find two outputs to drive them to desired values and then a third variable remains as a DOF to be corrected a posteriori. By taking into account the values of the tensor elements at the final location, the solution of the homogeneous linear system generated when the outputs are equal to zero and the invertibility of the matrix relating the output dynamics with the robot velocities, we find that it is feasible to design a square control system that corrects both longitudinal and lateral error, leaving the orientation as a DOF. The orientation error can be corrected in a second step considering that the robot uses a differential drive.

A. First-Step - Position Correction

Let us define the following sum of normalized tensor elements as outputs to be controlled:

$$\begin{aligned}\xi_1 &= T_{112} + T_{121}, \\ \xi_2 &= T_{212} + T_{221}.\end{aligned}\quad (5)$$

A robust tracking controller is proposed to take the value of both outputs to zero in a smooth way. Let $e_1 = \xi_1 - \xi_1^d$ and $e_2 = \xi_2 - \xi_2^d$ be the corresponding tracking errors, where ξ_1^d and ξ_2^d are suitable sinusoidal references. Using the time derivatives of these errors and considering that the camera location coincides with the vertical axis of rotation of the robot ($\ell = 0$), we obtain the error system

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} -\frac{c\phi_1}{T_N^m} & T_{122} - T_{111} \\ -\frac{s\phi_1}{T_N^m} & T_{222} - T_{211} \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} - \begin{bmatrix} \dot{\xi}_1^d \\ \dot{\xi}_2^d \end{bmatrix}. \quad (6)$$

This system has the form $\dot{e} = \mathbf{D}(\mathbf{T}, \phi_1) \mathbf{u} - \dot{\xi}^d$, where $\mathbf{D}(\mathbf{T}, \phi_1)$ corresponds to the decoupling matrix and $\dot{\xi}^d$ represents a known feedforward term. We treat the tracking problem as the robust stabilization of the error system (6). A control law based on sliding mode control [14], which has been already applied in the context of visual control [15], is proposed as follows:

$$\mathbf{u}_{db} = \begin{bmatrix} v_{db} \\ \omega_{db} \end{bmatrix} = \mathbf{D}^{-1} \begin{bmatrix} \dot{\xi}_1^d - \kappa_1 \text{sign}(s_1) - \lambda_1 s_1 \\ \dot{\xi}_2^d - \kappa_2 \text{sign}(s_2) - \lambda_2 s_2 \end{bmatrix} \quad (7)$$

where $\kappa_1 > 0$, $\kappa_2 > 0$, $\lambda_1 > 0$, $\lambda_2 > 0$ are control gains and $s_1 = e_1$, $s_2 = e_2$ are the so-called sliding surfaces. Note that the control law depends on the orientation of the fixed auxiliary camera ϕ_1 . This parameter can be fixed to zero and any error with respect to the real value is tackled by the robust control. The control law uses the inverse of the decoupling matrix \mathbf{D} to compute the robot velocities, which presents a singularity when the robot reaches the target position. This entails the problem that the rotational velocity ω_{db} increases to infinity as the robot reaches the target. However, we propose the commutation to a direct sliding mode controller when $\det(\mathbf{D})$ is near to zero in order to keep ω_{db} bounded. This kind of controller has been studied for output tracking through singularities [16]. For our case, a bounded sliding mode controller is

$$\mathbf{u}_b = \begin{bmatrix} v_b \\ \omega_b \end{bmatrix} = \begin{bmatrix} k_v \text{sign}(s_1) \\ -k_\omega \text{sign}(s_2 (T_{222} - T_{211})) \end{bmatrix} \quad (8)$$

where k_v and k_ω are suitable control gains.

B. Second-Step - Orientation Correction

Once position correction has been reached, we can use any single tensor element whose dynamics depends on ω and with desired final value zero to correct orientation. We select the dynamics $\dot{T}_{122} = -T_{112}\omega$. A suitable input ω is

$$\omega = \lambda_\omega \frac{T_{122}}{T_{112}}, \quad (9)$$

where $\lambda_\omega > 0$ is a control gain. This rotational velocity assigns the following dynamics to T_{122} , which is exponentially stable:

$$\dot{T}_{122} = -T_{112} \left(\lambda_\omega \frac{T_{122}}{T_{112}} \right) = -\lambda_\omega T_{122}.$$

Note that (9) never becomes singular because at the beginning of this step $T_{112} = -t_{y_1} \cos \phi_2$, and it tends to $-t_{y_1} \neq 0$ as final value. Although only a rotation is carried out in this second step, we keep the translational velocity v_b given in (8) in order to keep closed loop control along the whole motion.

IV. POSITION-BASED CONTROL FROM THE 1D TT

The elements of the 1D TT are very useful providing information of position and orientation of a camera [4]. We propose to make use of the information provided by the 1D TT to estimate the camera motion dynamically, according to the nonholonomic motion model (2). Once the robot pose is estimated, it can be used to control the robot in the Cartesian space.

Consider the problem of estimating the state $\mathbf{x}_k = (x_k, y_k, \phi_k)^T$ of the discrete model of the robot (2) by using measurements \mathbf{y}_k , which depend on the robot state through a nonlinear function h . It is assumed that the robot state and the measurements are affected by Gaussian noises \mathbf{m}_k and \mathbf{n}_k , respectively. The noisy system and measurement model can be expressed in compact form as follows:

$$\begin{aligned}\mathbf{x}_{k+1} &= f(\mathbf{x}_k, \mathbf{u}_k) + \mathbf{m}_k, \\ \mathbf{y}_k &= h(\mathbf{x}_k) + \mathbf{n}_k\end{aligned}\quad (10)$$

where it is accomplished $\mathbf{m}_k \sim N(0, \mathbf{M}_k)$, $\mathbf{n}_k \sim N(0, \mathbf{N}_k)$ and $E[\mathbf{m}_{k,i} \mathbf{n}_{k,j}^T] = 0$, with \mathbf{M}_k the state noise covariance and \mathbf{N}_k the measurement noise covariance. This estimation problem can be solved by a filtering approach using an Extended Kalman Filter (EKF), however, the property of observability must be ensured in order to achieve a consistent estimation.

A. Linear Observability from Measurements of the 1D TT

There are few works concerned about observability when an estimation based on Kalman filtering is applied. Some of them are [17] and [18]. To analyze our case, let us consider the linear approximation $(\mathbf{F}_k, \mathbf{G}_k, \mathbf{H}_k)$ of the system (2) in the time k , where

$$\mathbf{F}_k = \left. \frac{\partial f}{\partial \mathbf{x}_k} \right|_{\mathbf{x}_k = \hat{\mathbf{x}}_k^+, \mathbf{m}_k = 0}, \quad \mathbf{G}_k = \left. \frac{\partial f}{\partial \mathbf{u}_k} \right|_{\mathbf{x}_k = \hat{\mathbf{x}}_k^+}, \quad \mathbf{H}_k = \left. \frac{\partial h}{\partial \mathbf{x}_k} \right|_{\mathbf{x}_k = \hat{\mathbf{x}}_k^-, \mathbf{n}_k = 0}.$$

Due to the matrices \mathbf{F}_k and \mathbf{H}_k are changing at each instant time, observability may not be ensured, which affects the convergence properties of the estimation algorithm. As mention in [17], a system that is locally observable over every time segment $[t_k, t_{k+1}]$ in the interval $[t_0, t_{k+1}]$ will also be completely observable over the interval $[t_0, t_{k+1}]$. Then, the condition to accomplish for every k to ensure the system to be completely observable is

$$\text{rank} \left(\begin{bmatrix} \mathbf{H}_k^T & (\mathbf{H}_k \mathbf{F}_k)^T & \dots & (\mathbf{H}_k \mathbf{F}_k^{n-1})^T \end{bmatrix}^T \right) = n.$$

Because of the triangular form of the matrix \mathbf{F}_k , the rows of the observability matrix become linearly dependent. The only possibility of achieving the full rank condition is by building \mathbf{H}_k of full space. It can be done by taking three elements of the TT as outputs. By analyzing the Jacobian of each element of the tensor, we find that a suitable selection of measurements is T_{122} , T_{211} , T_{111} , in such a way that

$$\mathbf{H}_k = \begin{bmatrix} c\phi_1 c\hat{\phi} & c\phi_1 s\hat{\phi} & t_{y_1} c\hat{\phi} - \hat{t}_y c\phi_1 \\ -c\phi_1 s\hat{\phi} & c\phi_1 c\hat{\phi} & -t_{x_1} c\hat{\phi} + \hat{t}_x c\phi_1 \\ -s\phi_1 s\hat{\phi} & s\phi_1 c\hat{\phi} & t_{y_1} c\hat{\phi} + \hat{t}_x s\phi_1 \end{bmatrix}, \quad (11)$$

where $\hat{\phi} = \hat{\phi}_{k|k-1}$, $\hat{t}_x = -\hat{x}_{k|k-1} c\hat{\phi}_{k|k-1} - \hat{y}_{k|k-1} s\hat{\phi}_{k|k-1}$, $\hat{t}_y = \hat{x}_{k|k-1} s\hat{\phi}_{k|k-1} - \hat{y}_{k|k-1} c\hat{\phi}_{k|k-1}$, and t_{x_1} , t_{y_1} and ϕ_1 are constant values. The measurement matrix in (11) ensures local observability for every k even for some particular initial conditions, for instance $\phi_1 = 0$, in which case this matrix remains full rank due to the cosines in the main diagonal. It is worth emphasizing that the result is valid for normalized tensor elements, although we show the previous expressions for non-normalized elements for simplicity. Actually, observability from only one element of the tensor as measurement is ensured according to a nonlinear analysis [19]. In the same paper can be found a method for the EKF initialization using the information provided by the tensor.

B. Control Law using the Estimated Robot Pose

In this section, we assume that the robot pose is available, given by the EKF using three elements of the TT as measurements. The outputs to be controlled are the camera position

coordinates x_k and y_k . Consequently, the orientation (ϕ_k) is left as a DOF which is automatically corrected by tracking suitable desired trajectories. To take the value of both outputs to zero in a smooth way, we design a tracking controller. Let us define the tracking errors as $\xi_k^1 = x_k - x_k^d$, $\xi_k^2 = y_k - y_k^d$. Thus, the error dynamics $\xi_k = (\xi_k^1, \xi_k^2)^T$ obey the following difference equation:

$$\xi_{k+1} = \xi_k + T_s \begin{bmatrix} -s\phi_k & -lc\phi_k \\ c\phi_k & -ls\phi_k \end{bmatrix} \mathbf{u}_k - T_s \begin{bmatrix} \dot{x}_k^d \\ \dot{y}_k^d \end{bmatrix}. \quad (12)$$

We can see that the control inputs appear in the first order difference equation of each output. Then, the system (2) with outputs (x_k, y_k) has a vector relative degree $\{1,1\}$. Due to the sum of the indices of the system is less than the order of the system ($n = 3$) we have a first order zero dynamics, which corresponds to the DOF of the control system, the orientation ϕ_k . A static state feedback control law that achieves global stabilization of the system (12) is

$$\begin{bmatrix} \hat{v}_k \\ \hat{\omega}_k \end{bmatrix} = \frac{1}{\ell} \begin{bmatrix} -ls\hat{\phi}_k & lc\hat{\phi}_k \\ -c\hat{\phi}_k & -s\hat{\phi}_k \end{bmatrix} \begin{bmatrix} \hat{v}_k^1 \\ \hat{v}_k^2 \end{bmatrix}, \quad (13)$$

where $\hat{v}_k^1 = -k_1 \xi_k^1 + \dot{x}_k^d$ and $\hat{v}_k^2 = -k_2 \xi_k^2 + \dot{y}_k^d$. The error behavior will be exponentially stable iff $k_1 > 0$, $k_2 > 0$. Note that this input-output linearization via static feedback is valid for $\ell \neq 0$. Otherwise, a singular decoupling matrix is obtained. However, the case of having the camera shifted from the robot rotation axis over the longitudinal axis is a common situation. Orientation correction is simultaneously achieved by tracking a parabolic path in the Cartesian space, which is demonstrated in [19].

V. EXPERIMENTAL EVALUATION

Both proposed approaches have been tested in real-world experiments using the robot presented in Fig. 1(a). The camera acquires images of size 800×600 pixels. The 1D TT is estimated using the five-point method as described in Section II.B with estimated projection center $(x_0 = 404, y_0 = 316)$ as the only required calibration parameter. These experiments have been carried out using a tracking of features as implemented in the OpenCV library.

A. Image-based Approach

The experiment shown in this section corresponds to a trial of the image-based approach. Fig. 3(a) presents the resultant path, given by odometry, from the ground truth initial location $(-0.55 \text{ m}, -1.35 \text{ m}, -35 \text{ deg})$. The time to accomplished the pose-regulation task is almost 14 s. The execution time of the first step is set to 9.4 s through fixing a number of iterations in which the tracked references reach zero. Before that, we can see in Fig. 3(b) that the bounded sliding mode control law is applied when the singularity appears. Fig. 3(c) shows that the behavior of the outputs is always close to the desired one but with a small error. The reason of the remaining error is that our robotic platform is not able to execute commands at a frequency higher than 10 Hz, and consequently the performance of the sliding mode control is not the optimum. According to Fig. 4(a) the motion

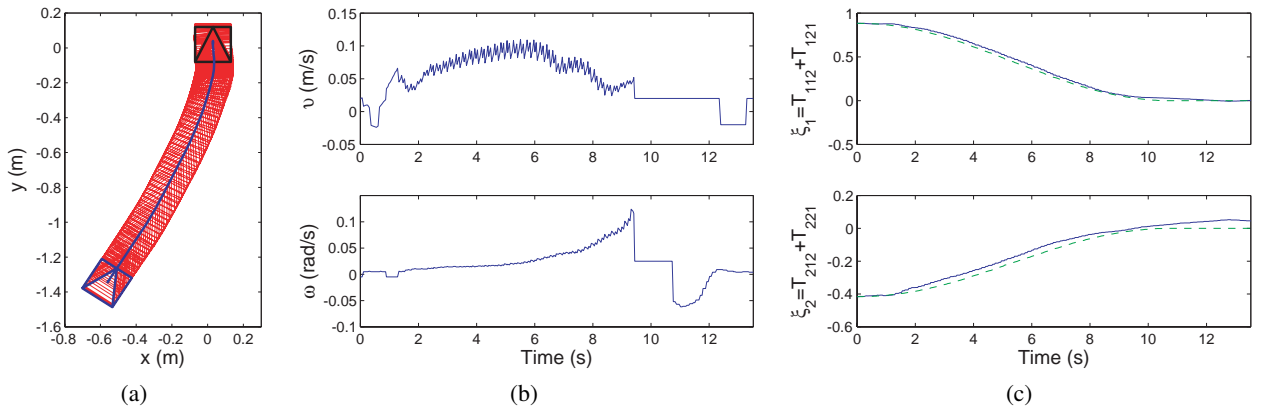


Fig. 3. Experimental results with the image-based approach. (a) Resultant path from the data given by the robot odometry. (b) Computed velocities. (c) Controlled outputs.

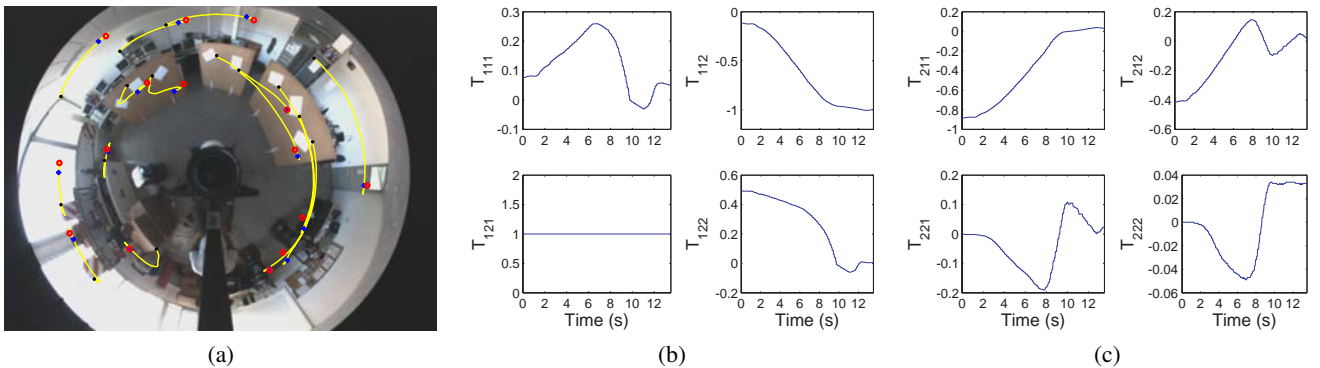


Fig. 4. Behavior of the visual measurements for the experiment using the image-based approach. (a) Motion of the image points. The marker “.” corresponds to the initial points, the marker “O” to the target points and the marker “+” are the points in the image at the end of the motion. (b) Evolution of the first four tensor elements. (c) Evolution of the second four tensor elements.

of the image points along the sequence does not exhibit a damaging noise, in such a way that the tensor elements evolve smoothly during the task, as presented in Fig. 4(b)-(c). The robot finally reaches the target with good precision.

B. Position-based Approach

For the evaluation of the position-based approach the sampling time T_s is set to 0.5 s. The distance from the camera to the rotation axis of the robot has been roughly set to $\ell = 10$ cm. The ground truth initial location is (-0.6 m, -1.8 m, 0 deg). Fig. 5(a) presents the resultant path, given by the estimated state of the robot, for one of the experimental runs. The figure also shows the reference path and the one given by odometry. It can be seen that the estimated path is closer to the reference than the path obtained from odometry. The duration of the pose-regulation task is fixed to 40 s, when the tracked references reach zero. Fig. 5(b) shows the behavior of the estimated state together with the tracked references for the position coordinates. According to Fig. 5(c) the robot velocities behave smoothly along the task, which represent an improved performance with respect to the image-based approach. Fig. 6(a) presents the well behaved motion of the image points along the sequence. The evolution of tensor elements is shown in Fig. 6(b). It is worth noting that the tensor estimation is not affected when the robot is reaching the target, i.e., there is no problem with the short baseline.

VI. CONCLUSIONS

In this paper, we have presented and evaluated experimentally two control schemes that rely on monocular vision to solve the pose-regulation problem of mobile robots. The proposed schemes are valid for vision systems obeying a central projection model, so that visibility constraint problems are avoided with the adequate sensor. In both proposed schemes, an adequate set of visual measurements are taken from the 1D trifocal tensor (TT). This tensor is estimated from bearing information of the visual features, which avoids the need of a complete camera calibration for any type of central camera and therefore, weakly-calibrated control schemes are obtained.

The properties of the 1D TT have been exploited in image-based (IB) and position-based (PB) schemes. The proposed IB scheme employs the direct feedback of elements of the 1D TT without commuting to any other approach during the whole task. This scheme is a two-step control law that is based on the sliding mode control technique. The proposed PB approach relies on the feedback of the estimated pose for control in the Cartesian space, with the benefits of reducing the dependence of the servoing on the visual data and facilitating the planning of complex tasks. We have shown that the 1D TT provides enough information to estimate the robot pose dynamically through a linear observability

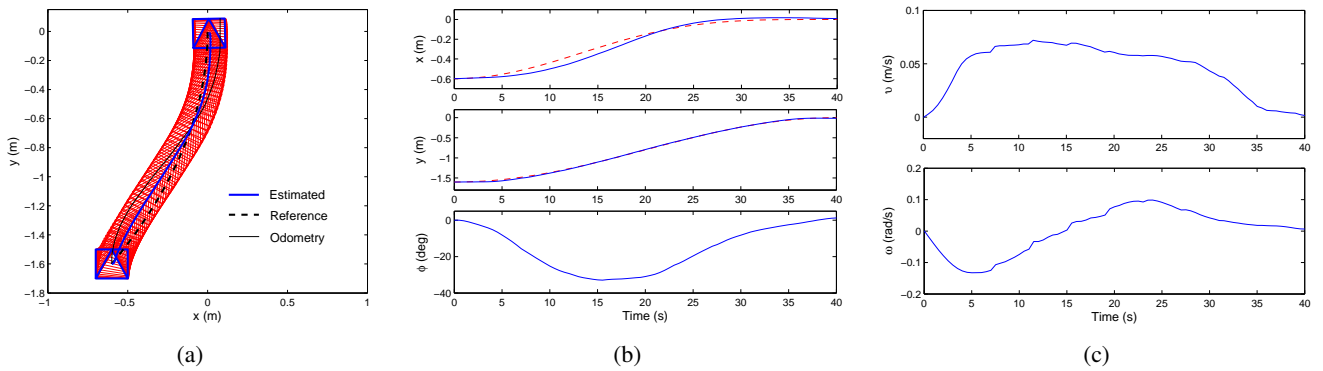


Fig. 5. Experimental results with the position-based approach. (a) Resultant path plotted using the estimated camera-robot state, although the reference path and the odometry are also shown. (b) Estimated camera-robot state. (c) Computed velocities.

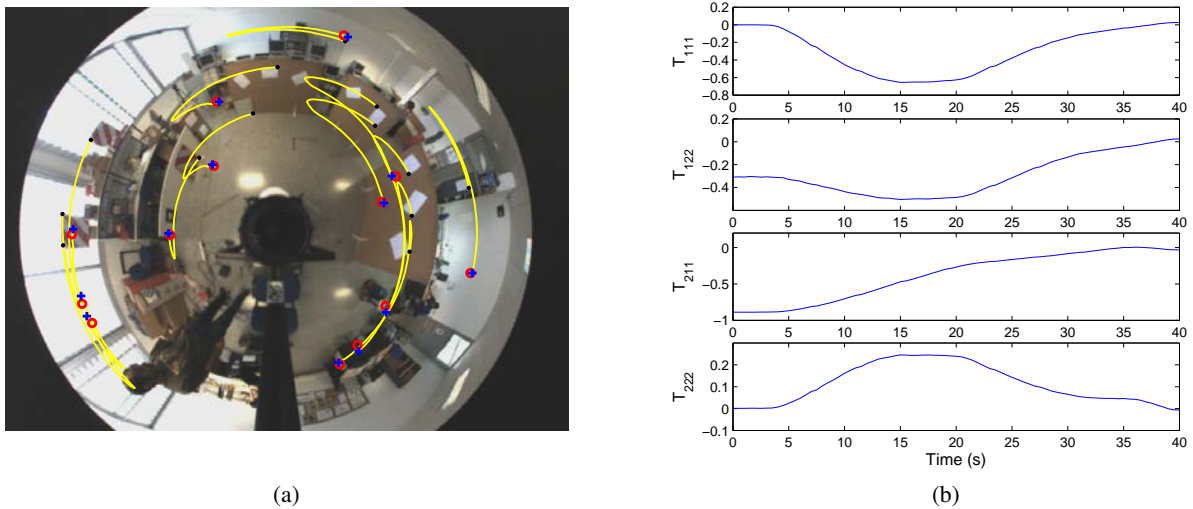


Fig. 6. Behavior of the extracted information from the images for the experiment with the position-based approach. (a) Motion of the point features. Initial points - “.”, target points - “O” and final points - “+”. (b) Four normalized tensor elements, three of them are taken as measurements.

analysis. The PB scheme is a single-step control law that corrects the robot pose using smooth robot velocities.

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