Quiz #5 solución

Las ecuaciones de CR son equivalentes a $d\alpha = 0$, donde $\alpha = fdz$. Esto es,

$$
\begin{aligned}
0 = dw \wedge dz &= [e^{i\varphi}(d\rho + i\rho d\varphi] \wedge [e^{i\theta}(dr + ird\theta)] \\
&= e^{i(\varphi+\theta)}[\rho_r dr + \rho_\theta d\theta + i\rho(\varphi_r dr + \varphi_\theta d\theta)] \wedge (dr + ird\theta) \\
&= e^{i(\varphi+\theta)}[-r\rho\varphi_r - \rho_\theta + i(r\rho_r - \rho\varphi_\theta)]dr \wedge d\theta,
\end{aligned}
$$

así que

$$\rho_\theta = -r\rho\varphi_r,\ r\rho_r = \rho\varphi_\theta\,.$$