4. Expresar en términos de $\cos\theta, \operatorname{sen}\theta$:

a) $\cos 3\theta$ b) $\operatorname{sen} 5\theta$ c) $\operatorname{sen} 7\theta$.

4b

$$\operatorname{sen} 5\theta = \operatorname{sen}^5\theta - 10\operatorname{sen}^3\theta\cos^2\theta + 5\operatorname{sen}\theta\cos^4\theta$$
$$= s^5 - 10 s^3 c^2 + 5 s c^4.$$

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$$= \frac{\left[(\cos\theta + i\operatorname{sen}\theta)^5 + (\cos\theta - i\operatorname{sen}\theta)^5\right]}{2i}$$

$i, -1, -i, 1, i,$
$1 \quad 2 \quad 3$
$1 \quad 0 \quad -1 \quad 0 \quad 1$
$c \qquad s$

$$= \operatorname{Im}(\cos\theta + i\operatorname{sen}\theta)^5$$

$$\operatorname{sen} 5\theta = \operatorname{Im}(e^{5i\theta}) = \operatorname{Im}((e^{i\theta})^5) = \operatorname{Im}((\cos\theta + i\operatorname{sen}\theta)^5)$$

$$= \operatorname{Im}\left(\cancel{c^5} + 5c^4(is) + \cancel{\binom{5}{2}c^3(is)^2} + \binom{5}{3}c^2(is)^3 + \cancel{\binom{5}{4}c(is)^4} + (is)^5\right)$$

$$= 5c^4 s - 10c^2 s^3 + s^5.$$

$\tan 5\theta = ?$ (expresar en términos de $\tan\theta$).

$\tan 2\theta = ?$

$\tan 3\theta = ?$

$\vdots$

$$\tan 5\theta = \frac{\operatorname{sen} 5\theta}{\cos 5\theta} = ?$$

$$\tan 2\theta = \frac{2t}{1-t^2} \quad \checkmark \qquad t = \tan\theta$$

$$\tan(\theta_1+\theta_2) = \frac{\operatorname{sen}(\theta_1+\theta_2)}{\cos(\theta_1+\theta_2)} = \frac{s_1 c_2 + s_2 c_1}{c_1 c_2 - s_1 s_2} = \boxed{\frac{t_1+t_2}{1-t_1 t_2}}$$

$\div c_1 c_2$

$$\tan 2\theta = \frac{2t}{1-t^2}$$

$$\boxed{\tan((n+1)\theta) = \frac{\tan n\theta + t}{1-(\tan n\theta)t}}$$

$$\tan 3\theta = \frac{2t/1-t^2 + t}{1 - \frac{2t}{1-t^2}t} = \frac{2t + t(1-t^2)}{1-t^2-2t^2} = \frac{3t-t^3}{1-3t^2} = \frac{t(3-t^2)}{1-3t^2}$$

$$\tan 5\theta = \frac{\frac{2t}{1-t^2} + \frac{t(3-t^2)}{1-3t^2}}{1 - \frac{2t^2(3-t^2)}{(1-t^2)(1-3t^2)}} = \frac{2t(1-3t^2)+(1-t^2)t(3-t^2)}{(1-t^2)(1-3t^2)-2t^2(3-t^2)} = \dots$$

5. Sea $z = \cos(\pi/50) + i\,\mathrm{sen}(\pi/50)$.

a) Demuestra que $1 + z^k + z^{2k} + \cdots + z^{99k} = 0$ si k no es un múltiplo de 100.

b) Encuentra el valor de $1 - z^k + z^{2k} + \cdots - z^{99k}$.

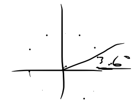
Obs. $z^{100} = 1$.

$$1 + z^k + z^{2k} + \cdots + z^{99k} =$$
$$= 1 + (z^k) + (z^k)^2 + \cdots + (z^k)^{99} =$$
$$= (z^k)^{100} - 1 / z - 1 = 0.$$

$z^k \neq 1$

$\Updownarrow$

$k \not\equiv 0 \pmod{100}$



$$\sum_N = \underbrace{1 + x + x^2 + \cdots + x^N}_{\text{serie geom.}} = \begin{cases} \dfrac{x^{N+1} - 1}{x - 1}, & x \neq 1, \\ N+1, & \text{si } x = 1. \end{cases}$$

[si $|x| < 1$, y $N \to \infty$, $\sum_N \to \frac{1}{x-1}$].

Dem: $S = 1 + x + \cdots + x^N$

$$S - 1 = x + \cdots + x^N = x(1 + x + \cdots + x^{N-1})$$
$$= x(S - x^N)$$
$$= xS - x^{N+1}$$
$$S(1 - x) = -x^{N+1}$$
$$S = x^{N+1} / x - 1. \checkmark$$

$$(1 + \cdots + x^N)(x - 1) =$$
$$= \cancel{x} + \cancel{x^2} + \cdots + \cancel{x^N} + x^{N+1}$$
$$- 1 - \cancel{x} - \cancel{x^2} - \cdots - \cancel{x^N} = x^{N+1} - 1.$$
$$\Rightarrow 1 + \cdots + x^N = (x^{N+1} - 1)/(x - 1)$$

$x \neq 1$

5b. $S = 1 \overbrace{- z^k}^{w} + z^{2k} - \cdots - z^{99k} = 0$?

$\boxed{\overset{\text{def}}{w := -z^k}}$

$$1 + w + w^2 + w^3 + \cdots + w^{99} =$$
$$= \begin{cases} w^{100} - 1 / w - 1 & w \neq 1 & (A) \\ 100 & w = 1 & (B) \end{cases}$$

(A) $-z^k \neq 1 \iff z^k \neq -1 \iff \cdots$

$\Rightarrow w^{100} = (-z^k)^{100} = (z^{100})^k = 1$

$\Rightarrow S = 0$.

(B) $-z^k = 1 \iff z^k = -1 \iff$

$\iff e^{\pi k i/50} = e^{i\pi}$

$\iff k\pi/50 \equiv \pi \pmod{2\pi}$ $\quad / \cdot \frac{50}{\pi}$ $\quad \frac{2\cancel{\pi} \cdot 50}{\cancel{\pi}} = 100$

$k \equiv 50 \pmod{100}$

$k = \pm 50, \pm 150, \ldots$