1. Encuentra el eje, vértice y foco de la parábola $\underbrace{4x^2 + 12xy + 9y^2}$ $+ 13x + 28y + 17 = 0$.

1) Rotación: $S = \begin{pmatrix} 4 & 6 \\ 6 & 9 \end{pmatrix}$

valores propios: $0 = \det(S - \lambda I) = \det\begin{pmatrix} 4-\lambda & 6 \\ 6 & 9-\lambda \end{pmatrix} =$

$= (4-\lambda)(9-\lambda) - 36 = \lambda^2 - 13\lambda = 0$ $\begin{cases} \lambda = 0 & \leftarrow \text{parabóla} \\ \lambda = 13 \end{cases}$

vect. prop: $\boxed{\lambda_1 = 0}$ $\begin{pmatrix} 4 & 6 \\ 6 & 9 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4x + 6y \\ 6x + 9y \end{pmatrix} \begin{matrix} = 0 \\ = 0 \end{matrix}$

$4x + 6y = 0$, $x = 1$, $y = -\frac{2}{3}$ $\Rightarrow v_1 = (1, -2/3) \Big/ \sqrt{1 + \frac{4}{9}} = \frac{3}{\sqrt{13}}(1, -\frac{2}{3}) = \boxed{\frac{(3, -2)}{\sqrt{13}}}$ v_1

$\boxed{\lambda_2 = 13}$ $\begin{pmatrix} -9 & 6 \\ 6 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 0 \Rightarrow -9x + 6y = 0 \quad / \div 3$

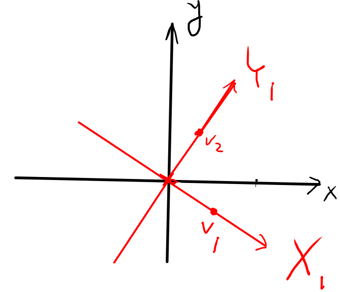
$-3x + 2y = 0$

$x = 2, \; y = 3$

$\boxed{v_2 = (2, 3) / \sqrt{13}}$

$$(x,y) = X_1 v_1 + Y_1 v_2 = \left(\frac{3X_1, -2X_1}{\sqrt{13}}\right) + \left(\frac{2Y_1, 3Y_1}{\sqrt{13}}\right)$$

$$= \Big(\underbrace{\frac{3X_1 + 2Y_1}{\sqrt{13}}}_{=\, x}, \underbrace{\frac{-2X_1 + 3Y_1}{\sqrt{13}}}_{=\, y}\Big)$$



$$\boxed{\begin{cases} x = \dfrac{3X_1 + 2Y_1}{\sqrt{13}} \\[2mm] y = \dfrac{-X_1 + 3Y_1}{\sqrt{13}} \end{cases}} \Rightarrow$$

$$R = \frac{1}{\sqrt{13}} \begin{pmatrix} 3 & 2 \\ -2 & 3 \end{pmatrix}$$

$$R^{-1} = \frac{1}{\sqrt{13}} \begin{pmatrix} 3 & -2 \\ 2 & 3 \end{pmatrix}$$

$$\Rightarrow \begin{cases} X_1 = \dfrac{3x - 2y}{\sqrt{13}} \\[2mm] Y_1 = \dfrac{2x + 3y}{\sqrt{13}} \end{cases}$$

Prop: Sea S matriz simétrica, v_1, v_2 vect. prop. de $\lambda_1 \neq \lambda_2 \Rightarrow (v_1, v_2) = 0$

Dem:

$$\begin{aligned} Sv_1 &= \lambda_1 v_1 \\ Sv_2 &= \lambda_2 v_2 \end{aligned} \Rightarrow \begin{aligned} (v_2, Sv_1) &= \lambda_1 (v_1, v_2) \\ (v_1, Sv_2) &= \lambda_2 (v_1, v_2) \end{aligned} \; -$$

$$0 = \underbrace{(\lambda_1 - \lambda_2)}_{\neq 0} \underbrace{(v_1, v_2)}_{= 0}. \quad \blacksquare$$

Cambio de variables $(x,y) \longmapsto (X_1, Y_1)$

$$0 = 4x^2 + 12xy + 9y^2 + 13x + 28y + 17 =$$

$$= 4\,\frac{9X_1^2 + 4Y_1^2 + 12X_1Y_1}{13} + 12\,\frac{-6X_1^2 + 6Y_1^2 + 5X_1Y_1}{13}$$

$$+ 9\,\frac{4X_1^2 + 9Y_1^2 - 12X_1Y_1}{13} + \sqrt{13}\,(3X_1 + 2Y_1) + \frac{28}{\sqrt{13}}(-2X_1 + 3Y_1) + 17 =$$

$$= \frac{1}{13}\cancel{(36 + 36 - 72)}\,X_1^2 + \frac{1}{13}\cancel{(9\cdot(-12) + 12\cdot 5 + 4\cdot 12)}\,X_1Y_1$$

$$+ \frac{81 + 12\cdot 6 + 16}{13}\,Y_1^2 + \left(\cancel{\frac{13\cdot 3}{\sqrt{13}} - \frac{56}{\sqrt{13}}}\right)X_1 + \left(\frac{26 + 84}{\sqrt{13}}\right)Y_1 + 17 = 0$$

$$-\frac{17}{\sqrt{13}} \qquad\qquad \frac{110}{\sqrt{13}}$$

Tanslación

$$\underbrace{13Y_1^2 + \frac{110}{\sqrt{13}}Y_1}_{\parallel} - \frac{17}{\sqrt{13}}X_1 + 17 = 0$$

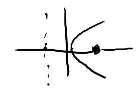
$$13\left(Y_1^2 + \frac{110}{13\sqrt{13}}Y_1\right)$$

$$\parallel$$

$$13\left[\left(\underbrace{Y_1 + \frac{55}{\underbrace{13\sqrt{13}}_{b}}}_{Y}\right)^2 - \left(\frac{55}{13\sqrt{13}}\right)^2\right] - \frac{17}{\sqrt{13}}X_1 + 17 = 0$$

$$13Y^2 - \frac{17}{\sqrt{13}}X_1 + 17 - \left(\frac{55}{13}\right)^2 = 0$$

$$\boxed{Y^2 = 4pX}$$



$$Y^2 = \underbrace{\frac{17}{13\sqrt{13}}X_1 - \frac{17}{13} + \left(\frac{55}{13}\right)^2\frac{1}{13}}_{4pX} = \underbrace{\frac{17}{13\sqrt{13}}}_{4p}\underbrace{\left(X_1 + \overbrace{\frac{13\sqrt{13}}{17}\left(-\frac{17}{13} + \left(\frac{55}{13}\right)^2\frac{1}{13}\right)}^{a}\right)}_{X}$$

$$X = X_1 + a$$

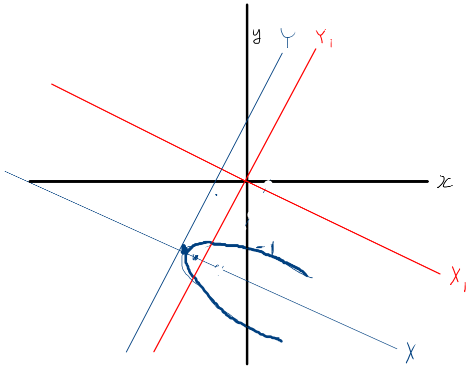
$$Y = Y_1 + b$$

$$Y^2 = 4pX$$

$$p = \frac{17}{4\cdot 13\sqrt{13}} = .1$$

$$\boxed{\begin{array}{l} a \approx 0.2 \\ b \approx 1,2 \end{array}}$$

Resumen



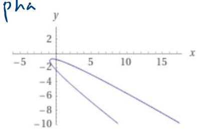
$X = Y = 0 \Leftrightarrow X_1 = -a, \; Y_1 = -b$

$= -.2 \qquad = -1.2$

$Y^2 = 4pX$

$$\begin{cases} \text{eje} : Y = 0 \\ \text{foco} : X = p \approx .1, \; Y = 0. \\ \text{vértice} : X = Y = 0 \end{cases}$$

gráfica de Wolfram alpha



$$\boxed{\begin{aligned} X_1 &= X - a \\ Y_1 &= Y - b \end{aligned}}$$

eje: $Y = 0 \Leftrightarrow Y_1 + b = 0$

$$\boxed{\frac{2x + 3y}{\sqrt{13}} + b = 0}$$

$$2x + 3y + b\sqrt{13} = 0$$

Parcial 2: 18 nov.

Final: 9 dic.