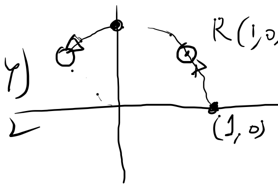
pR rotación ~ <u>diagonalizar</u> una <u>forma</u> <u>cuadrática</u>, pR ejemplo

$$Q(x,y) = 3x^2 + 4xy + y^2$$

O sea, encontrar una rotación $R(X,Y) = (x,y) = \overbrace{(aX - bY,\ bX + aY)}^{\text{rotación}}$

$$Q(R(X,Y)) = \lambda_1 X^2 + \lambda_2 Y^2 \qquad a^2 + b^2 = 1$$

<u>1era manera</u>: pedestre: $Q = 3(aX - bY)^2 + 4(aX - bY)(bX + aY) + (bX + aY)^2$



$$= -6ab + 4a^2 - 4b^2 + 2ab =$$

$$\begin{cases} -5ab + 4a^2 - b^2 = 0 & \text{①} \\ a^2 + b^2 = 1. & \text{②} \end{cases}$$

Obs: ① es "homogeneo" (si (a,b) es sol'n ent. $(\lambda a, \lambda b)$ también lo es.

Tomamos $b=1 \overset{①}{\Rightarrow} 4a^2 - 5a - 1 = 0 \Rightarrow \frac{5 \pm \sqrt{5^2+4\cdot 4}}{8} = \frac{5 \pm \sqrt{41}}{8}$

$\Rightarrow (a,b) = \lambda\left(\frac{5+\sqrt{41}}{8}, 1\right)$

$1 = a^2+b^2 = \lambda^2\left[\left(\frac{5+\sqrt{41}}{8}\right)^2 + 1\right]$

$\Rightarrow \lambda = 1 \Big/ \sqrt{\left(\frac{5+\sqrt{41}}{8}\right)^2 + 1}$

$\Rightarrow (a,b) = \frac{1}{\sqrt{\left(\frac{5+\sqrt{41}}{8}\right)^2+1}}\left(\frac{5+\sqrt{41}}{8}, 1\right).$

Cómo diagonalizar

$$Q(x,y) = 3x^2 + 4xy + y^2 = \left\langle \begin{pmatrix} x \\ y \end{pmatrix}, \underbrace{\begin{pmatrix} 3x+2y \\ 2x+y \end{pmatrix}}_{\underbrace{\begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}}_{S} \begin{pmatrix} x \\ y \end{pmatrix}} \right\rangle$$

Más sofisticado.

Buscamos vectores propios

$$S v_1 = \lambda_1 v_1, \quad S v_2 = \lambda_2 v_2 \; ; \quad \langle v_1, v_1 \rangle = \langle v_2, v_2 \rangle = 1, \quad \langle v_1, v_2 \rangle = 0$$

$\cancel{\lambda_1}$, $\lambda_2 = ?$

$$\det(S - \lambda I) = 0$$

$$\det \begin{pmatrix} 3-\lambda & 2 \\ 2 & 1-\lambda \end{pmatrix} = 0$$

$$(3-\lambda)(1-\lambda) - 4 = 0$$

$$\lambda^2 - 4\lambda + 3 - 4 = 0$$

$$\lambda^2 - 4\lambda - 1 = 0$$

$$(\lambda - 2)^2 - 4 - 1 = 0$$

$$(\lambda - 2)^2 = 5$$

$$\lambda - 2 = \pm\sqrt{5}$$

$$\boxed{\lambda = 2 \pm \sqrt{5}}$$

$$\left.\begin{aligned} \lambda_1 &= 2 + \sqrt{5} \\ \lambda_2 &= 2 - \sqrt{5} \end{aligned}\right\} \text{ hipérbola!}$$

$$\lambda_1 = 2+\sqrt{5}, \quad \begin{pmatrix} 3-2-\sqrt{5} & 2 \\ 2 & 1-2-\sqrt{5} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$$(1-\sqrt{5})x + 2y = 0, \quad x=1 \Rightarrow y = \frac{-1+\sqrt{5}}{2}$$

$$\boxed{v_1 = \left(1, \frac{-1+\sqrt{5}}{2}\right) \Big/ \sqrt{1+\left(\frac{-1+\sqrt{5}}{2}\right)^2}} \quad (\Rightarrow \|v_1\|^2 = 1)$$

$$\boxed{\lambda_2 = 2-\sqrt{5}}, \quad \begin{pmatrix} 3-2+\sqrt{5} & 2 \\ * & * \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow (1+\sqrt{5})x + 2y = 0, \quad x=1 \Rightarrow y = -\frac{1+\sqrt{5}}{2}$$

$$\boxed{v_2 = \left(1, -\frac{1+\sqrt{5}}{2}\right) \Big/ \sqrt{1+\left(\frac{1+\sqrt{5}}{2}\right)^2}}, \quad \|v_2\|^2 = 1$$

Teo: los vect. propios que corresponden a valores propios distintos de una matriz simétrica, son perpendiculares

$$\langle v_1, v_2 \rangle = 0.$$

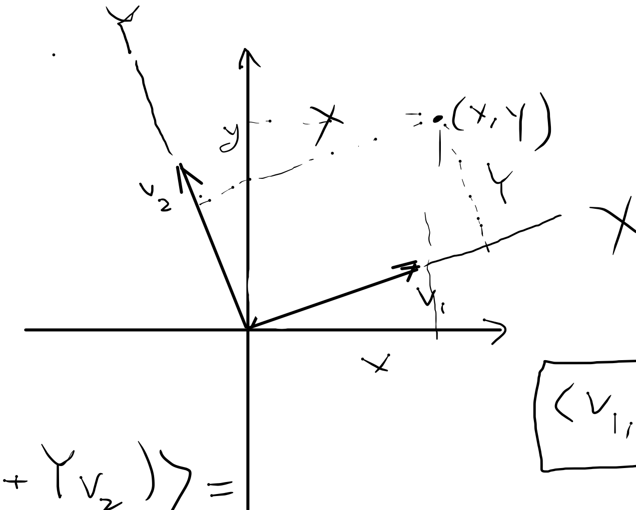
Checamos:

$$1^2 - \left(\frac{\sqrt{5}-1}{2}\right)\left(\frac{\sqrt{5}+1}{2}\right) =$$

$$= 1 - \frac{5-1}{4} =$$

$$= 1 - \frac{4}{4} = 0. \quad //$$

$$R = \begin{pmatrix} v_1 & v_2 \end{pmatrix}$$



$$\boxed{\langle v_1, v_2 \rangle = 0}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = X v_1 + Y v_2$$

$$Q\begin{pmatrix} x \\ y \end{pmatrix} = \left\langle \begin{pmatrix} x \\ y \end{pmatrix}, S\begin{pmatrix} x \\ y \end{pmatrix} \right\rangle =$$

$$= \langle X v_1 + Y v_2, S(X v_1 + Y v_2) \rangle =$$

$$= \langle X v_1 + Y v_2, X S v_1 + Y S v_2 \rangle$$

$$= \langle X v_1 + Y v_2, \lambda_1 X v_1 + \lambda_2 Y v_2 \rangle =$$

$$= \lambda_1 X^2 + \lambda_2 Y^2 = \left(\frac{X}{1/\sqrt{\lambda_1}} \right)^2 - \left(\frac{Y}{1/\sqrt{|\lambda_2|}} \right)^2$$

$$\lambda_1 > 0, \quad \lambda_2 < 0$$

semieje mayor