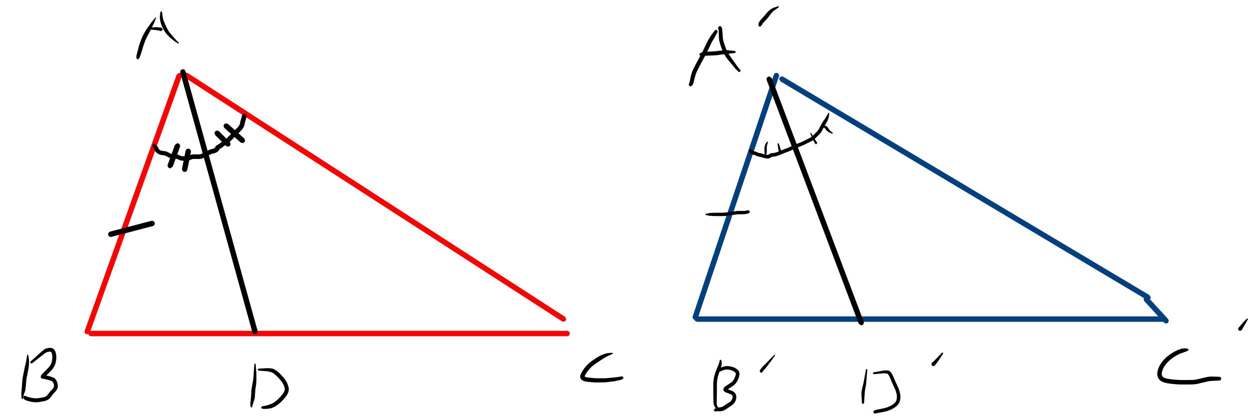
79. Suppose that an angle, its bisector, and one side of this angle in one triangle are respectively congruent to an angle, its bisector, and one side of this angle in another triangle. Prove that such triangles are congruent.

respectivamente



Dado:
- $\measuredangle BAD = \measuredangle DAC$
- $\measuredangle B'A'D' = \measuredangle D'A'C'$
- $\measuredangle BAC = \measuredangle B'A'C'$
- $AD = A'D'$
- $AB = A'B'$

P.D. $\triangle ABC \cong \triangle A'B'C'$

cosa 1, cosa 2, cosa 3 son iguales a cosa A, cosa B, cosa C respectivamente.

Problema: Encontrar todas las "ternas pitagóricas":

$a, b, c \in \mathbb{Z}$ tal que $a^2 + b^2 = c^2$. (E.g. $(3, 4, 5)$, $(5, 12, 13)$)

los núm enteros: $\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$

Por ejemplo

144 169

"primitivas" (sin factor común)

Idea: • $\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1 \Rightarrow x^2 + y^2 = 1$

$\frac{a}{c} = x$, $\frac{b}{c} = y$

núm racional

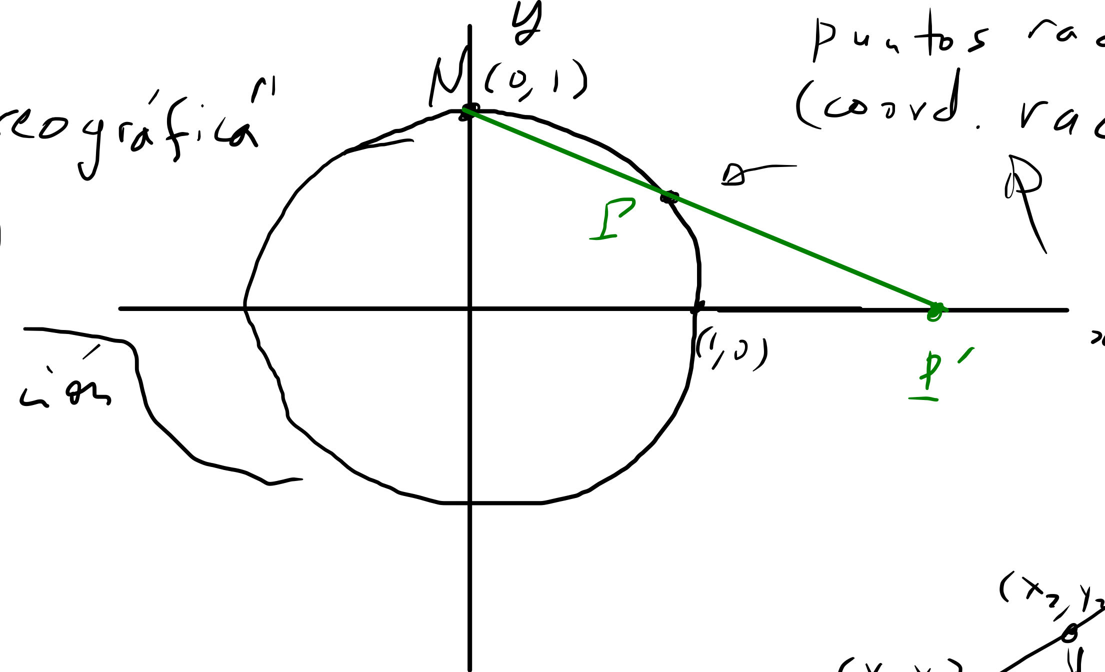
$x^2 + 3x - 7 = 0$

$\mathbb{Q}$ = racionales = fracciones = $\left\{ \frac{a}{b} \;\middle|\; a, b \in \mathbb{Z},\ b \neq 0 \right\}$

$\mathbb{Q} \supset \mathbb{Z}$

• usamos la "proyección estereográfica"

$P = (x, y) \longmapsto P' = (t, 0)$



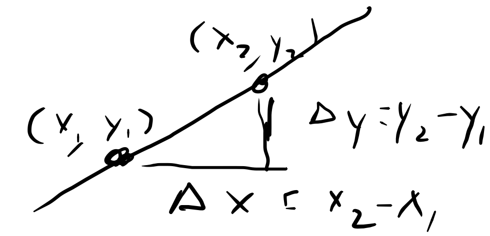
Meta: escribir t como función de (x, y).

Ej (5 mnts).

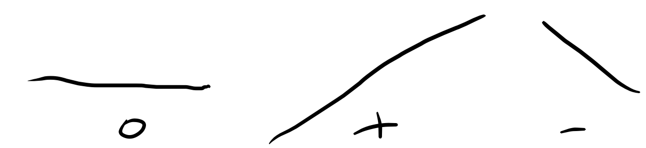
$$\boxed{t = \frac{x}{1-y}}$$

$$\frac{y-1}{x} = \frac{0-1}{t-0} \Rightarrow t = \frac{x}{1-y}$$

N, P   N, P'



pendiente $\overset{\text{def.}}{:=} \dfrac{\Delta y}{\Delta x}$



Teo: no importa cuales 2 puntos tomas sobre la recta para calcular su pendiente, siempre da el mismo número

Obs: si (x, y) son racionales $\Rightarrow t$ es racional!

• ¿Será cierto que la inversa de la proy. ester. también tiene esta propiedad?

Cuenta: expresar (x,y) en términos de t.

$$\begin{cases} t = \dfrac{x}{1-y} \\ x^2 + y^2 = 1 \end{cases} \Rightarrow x, y = ?$$

$$x = t(1-y) \Rightarrow [t(1-y)]^2 + y^2 = 1$$

$$t^2(1+y^2-2y) + y^2 = 1$$

$$y^2(t^2+1) - 2t^2y + (t^2-1) = 0$$

$$y = \frac{2t^2 \pm \sqrt{(2t^2)^2 - 4(t^2+1)(t^2-1)}}{2(t^2+1)} = \frac{2t^2 \pm \sqrt{\cancel{4t^4} - 4(\cancel{t^4}-1)}}{2(t^2+1)} =$$

$$= \frac{\cancel{2}t^2 \pm \cancel{2}}{\cancel{2}(t^2+1)} = \frac{t^2 \pm 1}{t^2+1} = \begin{cases} 1 \\ \dfrac{t^2-1}{t^2+1} \end{cases}$$

$$x = t(1-y) = t\left(1 - \frac{t^2-1}{t^2+1}\right) = t\left(\frac{\cancel{t^2}+1-\cancel{t^2}+1}{t^2+1}\right) = \frac{2t}{t^2+1}$$

Resp: $$P = (x,y) = \left(\frac{2t}{t^2+1}, \frac{t^2-1}{t^2+1}\right)$$

Obs (misma): si t es racional $\Rightarrow$ (x, y) son racionales!

- $t = \frac{m}{n} \in \mathbb{Q} \Rightarrow x = \frac{a}{c} = \frac{2t}{t^2+1} = \frac{2(m/n)}{(m/n)^2+1} = \frac{2m}{\cancel{n}\left(\frac{m^2+n^2}{n^{\cancel{2}}}\right)} = \boxed{\frac{2mn}{m^2+n^2}}$

- $y = \frac{b}{c} = \frac{\frac{m^2}{n^2}-1}{\frac{m^2}{n^2}+1} = \boxed{\frac{m^2-n^2}{m^2+n^2}}$

Teorema: Tomamos: $a = 2mn$, $b = m^2 - n^2$, $c = m^2 + n^2$.
cuando m, n son enteros con $m > n > 0$, (a, b, c) es una terna pitagórica, y así se obtiene todas las ternas pitagóricas.

Ej:

$2, 1 \Rightarrow 4, 3, 5$

$3, 2 \Rightarrow 12, 5, 13$

$3, 1 \Rightarrow 6, 8, 10$

$5, 2 \Rightarrow 20, 21, 29$