$$\varphi \in \mathrm{Ann}\{ v^{\otimes n} \mid v \in V \} \subseteq [S^n(V)]^*.$$

d

<u>P.D.</u> $\varphi = 0$.

- $\mathrm{Sen}\left\{ e_1^{m_1} \odot \cdots \odot e_d^{m_d} \;\middle|\; \sum_{i=1}^{d} m_i = n,\ 0 \le m_i \le n \right\}$ base de $S^n(V)$

$$\cancel{V^{\otimes n} = \mathrm{Span}\{ e_{i_1} \otimes \cdots \otimes e_{i_n} \mid 1 \le i_k \le d \}}$$

$$\cancel{d^n}$$

$$\cancel{V^{\otimes 2} = \{ e}$$

- ¿Cual es la base dual a esta base?

$\{ \varphi_1^{m_1} \odot \cdots \cdots \odot \varphi_d^{m_d} \mid \cdots \}$ donde

$\varphi_1, \dots\dots, \varphi_d$ la dual $e_1, \dots, e_d$

$\quad \parallel \qquad\qquad \parallel$

$\quad x_1, \qquad , x_d$

$$v = \sum x_i e_i \xmapsto{\varphi_i} x_i$$

$$\Rightarrow \{ x_1^{m_1} \cdots\cdots x_d^{m_d} \mid \cdots \}$$

2

$$x_1^2 \, x_2^3 \, x_3^7 \in [S^{12}(\mathbb{C}^3)]^*$$

H_{12}

$$\mathbb{C}[x_1, x_2, x_3] = \bigoplus H_n$$

H_n = pol. homog. de grado n.

<u>Lemma:</u> $\varphi \in H_{12}$ es $= 0$ ssi $\varphi(v) = 0 \; \forall v \in V$, si el campo de coefs es inf.

<u>más general:</u> si $p(x_1, \ldots, x_d) \in K[x_1, \ldots, x_d]$ y $p(a) = 0 \; \forall a \in K^d$, y K ~~es~~ tiene más elementos que $\cancel{d}$ grado(p) $\Rightarrow p = 0$

$$p(x_1, \ldots, x_d) = \sum a_{\cdots} \, x_1^{\cdot\cdot} \, x_2^{\cdot\cdot} \cdots =$$

$$= \sum q_i(x_1, \ldots x_{d-1})(x_d)^i \, .$$

para $x_1, \dots, x_{d-1}$ fijos es un pol de ~~de grad~~ ~~tan~~ una variable con ~~más~~ raíces que su grado

$\Rightarrow$ el pol es 0 $\Rightarrow a_i(x_1, \dots, x_{d-1}) = 0$ para ~~de~~ todo $x_1, \dots, x_{d-1} \in K$

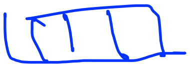
$\Rightarrow$ todo es puro 0.
ind

<u>Ejemplo:</u> <u>$n=4$</u> $V = \mathbb{C}^d$

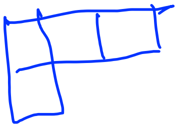
$$V^{\otimes 4} \underset{\varphi}{=} \bigoplus_{\lambda} V_\lambda \otimes S_\lambda(V)$$

bajo $S_4 \times GL(V)$

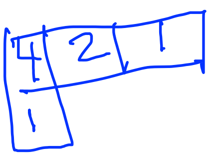
$$\left\{ \ydiagram{4}, \ydiagram{3,1}, \ydiagram{2,2}, \ydiagram{2,1,1}, \ydiagram{1,1,1,1} \right\} = \hat{S}_4$$

$\lambda =$  , $V_\lambda = \mathbb{C} =$ trivial.

$V_\lambda \otimes \mathbb{S}_\lambda(V) = S^4(V)$

$\lambda =$  , $V_\lambda = \mathbb{C}[S_4]c_\lambda$

$\dim V_\lambda = \frac{4!}{2 \cdot 4} = 3$



$V_\lambda \otimes \mathbb{S}_\lambda(V)$ como rep'n de $GL(V)$ es la suma de 3 irred

Ej:

$V_\lambda \cong V_\lambda^*$

sin calcular explicitamente los charac.

$\chi_{V^*}(g) = \chi_V(g^{-1})$

g conj g^{-1}.

- como podemos caracterizar estos tensores?
- hay manera (natural) de descomponerlo en 3 irreducibles isomorfos?

• ~~Sea~~ Será uno de estos irreducibles justo los tensores tipo curvatura en geom. dif.?

<u>curvi</u> $T \in V^{\otimes 4}$ t.q.

- $(12)T = (34)T = -T$
- $(13)(24)T = T$
- $\big((123)+(321)+e\big)T = 0$

$\ker\begin{pmatrix} e+(12), \\ e+(34) \end{pmatrix}$

$$c_\lambda = \big(e+(12)+(23)+(13)+(123)+(321)\big)\big(e-(14)\big)$$

1	2	3
4		

¿

$A = \mathbb{C}[S_4]$, ~~$A c$~~ $A c_\lambda = \{ a \in \mid a s_\lambda = 0$

s_λ

~~$V^{\otimes}$~~

Aquí lo dejamos.

$$V^{\otimes 4} c_\lambda \cong S_\lambda(V)$$

$$= \ker\left(\Lambda^3(V) \otimes V \to \Lambda^4(V)\right)$$

$$\lambda = \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & \multicolumn{2}{c}{} \\ \cline{1-1}\end{array}$$

$\boxed{S_3}$

$$\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & \multicolumn{1}{c}{} \\ \cline{1-1}\end{array}$$

, + otros 5 tableaus.

$$\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & \multicolumn{1}{c}{} \\ \cline{1-1}\end{array}$$

$$V_\lambda \otimes S_\lambda(V) \approx$$

$$(\qquad S_\lambda(V) \oplus S_\lambda(V)$$

2

$\omega \in \Lambda^2(T^*M)$, g

$\boxed{\nabla \cdot \omega} \in (T^*M)^{\otimes 3}$

3 tensor

quasihermitian

$\nabla\omega \in \Lambda^3 \iff$ nearly Kähler

⋮