Rep de grupos de Lie - generales 2/abr/2020

SU_2 explícito $V_0, V_1, \ldots$

- $G \rightsquigarrow \mathfrak{g} = T_e(G) \leftarrow$ alg de Lie

e.g. $G = GL_n(\mathbb{R})$, $\mathfrak{g} = T_e(G) \approx Mat_n(\mathbb{R})$

$\cap$

$Mat_n(\mathbb{R})$ $[A, B] = AB - BA$

- $\rho : G \longrightarrow H$ un homo de grupos Lie

$d\rho(e) : T_e(G) \longrightarrow T_e(H)$ $\rho'[X, Y] = [\rho' X, \rho' Y]$

$\rho' : \mathfrak{g} \rightarrow \mathfrak{h}$ homo. de alg. de Lie

$$\rho \rightsquigarrow \rho'$$

homo. de grupos — homo de alg.

<u>Teo:</u> ρ está determinado por ρ', para G arco conexo.

ie $\rho_1, \rho_2 : G \to H$, G conexo, $\rho_1' = \rho_2'$

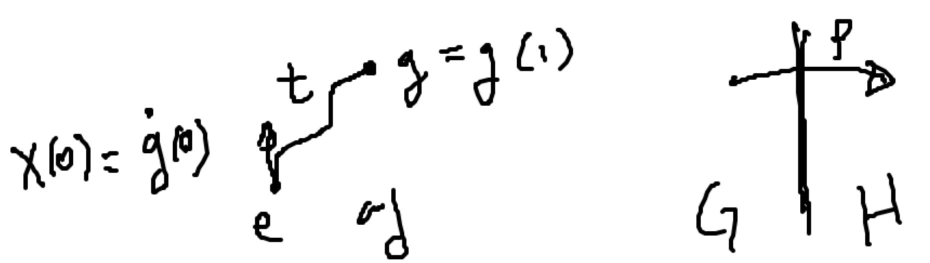
$$\Longrightarrow \rho_1 = \rho_2$$

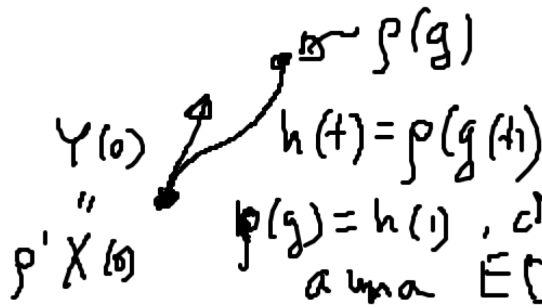
$\boxed{\text{idea}}$ teo. de unic. de sol'n de EDO

$$\dot{y} = f(t, y)$$

$$y = y(t) \in \mathbb{R}^n$$

$$y(0) = y_0 \qquad \dot{y} = y^2$$



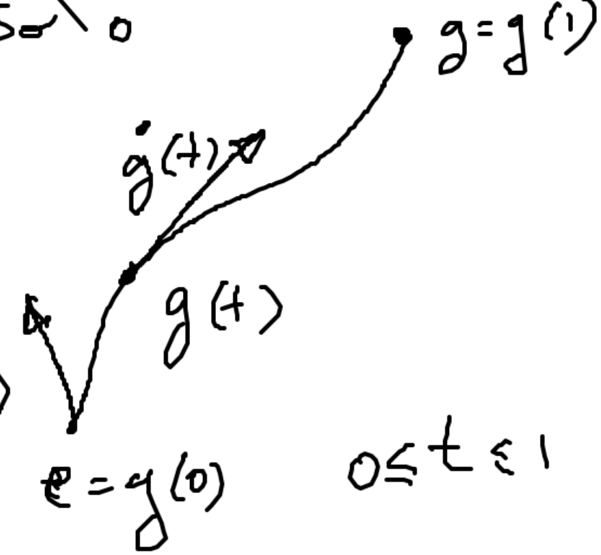


$\rho(g) = h(1)$, donde $h(t)$ es una sol a una EDO .....

$h(t) = \rho(g(t))$ sat. una EDO, que sólo depende de $g(t)$, y $Y(0) = \rho'(X(0))$

$X(0) = \dot g(0)$

$\dot h(t) = h(t)\, Y(t)$ $\qquad \cancel{(X)}$ $\qquad g^{-1}(t)\, \dot g(t) = X(t)$



$0 \le t \le 1$

$$\begin{array}{c} \dot g(t) \in T_{g(t)} G \\ \| \\ \dot g(t) = g(t)\, X(t) \\ \Downarrow d\rho \end{array}$$

$\dot h(t) = h(t)\, Y(t)$, donde $Y(t) = \rho' X(t)$, $h(t) = \rho(g(t))$

$L_g : G \to G, \quad x \to gx$

$$\begin{array}{ccc} dL_g : T_x G & \to & T_{gx} G \\ \cup & & \\ X & \longmapsto & gX \end{array}$$