Armónicos esféricos. 26/3/20

①

$S^2 \subset \mathbb{R}^3$, $f : S^2 \to \mathbb{C}$

?

$$SO_3 \circlearrowright L^2(S^2) = \bigoplus_{m=0}^{\infty} H_m$$

H_m = pol. homog. $p(x)$ de grado m en $\mathbb{R}^3$, armónicos, restringidos a S^2. $(\Delta_{\mathbb{R}^3} P = 0)$.

sub. esp. inv. irred. de dim $2m+1$.

$w = 0$

$$U(1) = \left\{ \begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta} \end{pmatrix} \right\}$$

$$\begin{array}{ccccc} & & U(1) & & SO_2 \\ & & \cap & & \cap \\ \{\pm I\} & \to & SU_2 & \xrightarrow[2:1]{Ad} & SO_3 \\ & & \uparrow & & \\ & & V_0, V_1, V_2, \dots & & \end{array}$$

dim: 1, 2, 3, ...

Def: los pesos de una repn de SU_2 son los pesos de su restricción a U_1

Pesos de V_d: $(-d, -d+2, \dots, d)$

dim $V_d = d+1$

$\boxed{\text{Teo repns de } U_1 = \{ w \mid w \in \mathbb{C}, |w| = 1 \}}$

Irred $\rho_n(w) = w^n$, $n \in \mathbb{Z}$, (repaso)

$V = \text{rep'n de } U_1 = \bigoplus_{i=1} \rho_{n_i} \Rightarrow \underbrace{n_1, n_2, \dots}_{\text{los pesos de } V} \in \mathbb{Z}$

$\chi_V = \sum_i w^{n_i}$

$$SO_2 = \left\{ \begin{pmatrix} \cos t & \text{sen} & 0 \\ -\sin & \cos & 0 \\ 0 & 0 & 1 \end{pmatrix} \right\} \quad z = e^{it}$$

$$U_1 \xrightarrow[2:1]{Ad} SO_2 \qquad z = Ad(w) = w^2$$

$$w \in \bigcirc \ni 1 \to \bigcirc \ni z$$

Lema: Si una rep ρ de SO_2 tiene pesos $(n_1, n_2, \dots)$, entonces los pesos de $\rho \circ Ad$ son $(2n_1, 2n_2, 2n_3, \dots)$

Dem: $\rho_n(z) = z^n \Rightarrow (\rho_n \circ Ad)(w) = Ad(\rho_n(w)) = w^{2n}$

V_{2m} : $(-2m, -2m+2, \dots, 2m)$ 3

$\shortparallel$

$H_m \circ Ad$:

H_m : $(-m, -m+1, \dots, m)$

$Y_{m,-m},\ Y_{m,-m+1},\ \dots,\ Y_{m,m}$

$Y_{m,0} = P_m(x_3)$  $\uparrow x_3$ SO_2-inv

pol. de Legendre

$$Y_{m,k} = \underbrace{(x_1 - i x_2)^k}_{u^k} P_m^{(k)}(x_3)$$

$U_m \, (\approx H_m) = \overline{U}_m$ prob. 6

[V] p.92

V = esp. vect $/\mathbb{C}$

$\overline{V} = ?$ $\boxed{\lambda \cdot v = \bar{\lambda} v}$ mismo conj.

dist. est. compleja

prob. 6 (cont.) 4

$$[\overline{T}] = \overline{[T]}$$

$$\boxed{\overline{\rho_n} = \rho_{-n}}$$

pesos

$$V \xrightarrow{T} W \qquad (n_1, n_2, \dots)$$

$$\big\downarrow \text{conj}$$

$$\overline{V} \xrightarrow{\overline{T}} \overline{W} \qquad (-n_1, -n_2, \dots)$$

$$\overline{T}(v) := T(v)$$

$$U_m \sim (-m, -m+1, \dots, m)$$

$$\overline{U}_m \sim (m, m-1, \dots, -m)$$

1era parte

$$\overline{Y_{m,k}} = Y_{m,-k} \begin{pmatrix}\text{salvo}\\ \text{escalar}\end{pmatrix}$$

$$\rho, \underbrace{\rho^*, \overline{\rho}}_{\text{equiv}}$$

equiv a ρ es unit.

$$T = \begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \end{pmatrix} \quad \begin{matrix} v_1 \\ \vdots \\ v_n \end{matrix}$$

$$\overline{T} = \begin{pmatrix} \overline{\lambda}_1 & & \\ & \ddots & \end{pmatrix}$$

$$U_m \xrightarrow{\sim} \overline{U}_m$$

$$f \mapsto \overline{f}$$

iso. concreto